# CHAPTER 3 AM Surds

Note: No mixed number with surds.
✓ $\frac{5}{3}\sqrt{x}$ ✗ $1\frac{2}{3}\sqrt{x}$

1)

| Rules | | Examples |
|---|---|---|
| Multiplication | $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$<br>$x\sqrt{a} \times y\sqrt{b} = xy\sqrt{ab}$<br>Number × Number, Surd × Surd | $\sqrt{5} \times \sqrt{3} = \sqrt{15}$<br>$6\sqrt{5} \times 8\sqrt{3} = 48\sqrt{15}$ |
| Multiplying Same surds | $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$<br>$\sqrt{a} \times \sqrt{a} \times \sqrt{a} = (\sqrt{a})^3$<br>$b\sqrt{a} \times b\sqrt{a} = (b\sqrt{a})^2 = b^2a$ | $\sqrt{5} \times \sqrt{5} = 5$<br>$\sqrt{5} \times \sqrt{5} \times \sqrt{5} = 5\sqrt{5}$<br>$3\sqrt{5} \times 3\sqrt{5} = 9(5) = 45$ |
| Division | $\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ | $\sqrt{10} \div \sqrt{2} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$ |
| Addition | $a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$ | $8\sqrt{5} + 3\sqrt{5} = 11\sqrt{5}$ |
| Subtraction | $a\sqrt{x} - b\sqrt{x} = (a-b)\sqrt{x}$ | $8\sqrt{5} - 3\sqrt{5} = 5\sqrt{5}$ |

## 2) Rationalising

- "Get rid" of surds in the denominator
- Apply special identity: $(a+b)(a-b) = a^2 - b^2$

Eg: Rationalise the following

$$\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

$$\frac{5}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{6(3)} = \frac{5\sqrt{3}}{18}$$

$$\frac{5}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{5(\sqrt{3}-2)}{(\sqrt{3})^2 - 2^2}$$
$$= \frac{5\sqrt{3}-10}{3-4}$$
$$= 10 - 5\sqrt{3}$$

$$\frac{7}{2-5\sqrt{3}} \times \frac{2+5\sqrt{3}}{2+5\sqrt{3}} = \frac{7(2+5\sqrt{3})}{2^2 - (5\sqrt{3})^2}$$
$$= \frac{14+35\sqrt{3}}{4-75}$$
$$= -\frac{14+35\sqrt{3}}{71}$$

## 3) Simplifying surds

- Use perfect squares: 4, 9, 16, 25, 36, 49...

Eg: $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$\sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$

$2\sqrt{50} = 2\sqrt{25 \times 2} = 2 \times 5\sqrt{2} = 10\sqrt{2}$

TIP

To check if you simplified correctly, type the question and your final answer in the calculator and check if the values match.

# Types of Surds Questions and Steps to tackle them :

## Type A : Simplify and / or rationalise expressions involving surds

① Simplify all surds to the simplest form (eg : $\sqrt{32} = 4\sqrt{2}$ )

Tip : Pick the smallest root that you see in the question and change all to that root.

② Combine all the like terms (those with same root)

③ Combine fractions into one single fraction by making same denominator.

④ For fractions, check denominator : If there are surds, rationalise.

Eg : Simplify $\sqrt{27} + 2\sqrt{48} - \frac{10}{\sqrt{3}}$

$\sqrt{27} + 2\sqrt{48} - \frac{10}{\sqrt{3}}$

① $= 3\sqrt{3} + 2\times4\sqrt{3} - \frac{10}{\sqrt{3}}$

② $= 3\sqrt{3} + 8\sqrt{3} - \frac{10}{\sqrt{3}}$

④ $= 11\sqrt{3} - \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ← we can rationalise for just one specific term when needed

TIP

$= 11\sqrt{3} - \frac{10\sqrt{3}}{3}$

$= 11\sqrt{3} - \frac{10}{3}\sqrt{3}$

$= \frac{23}{3}\sqrt{3}$

Eg : Simplify $\frac{\sqrt{3}+2}{\sqrt{3}} - \frac{4}{\sqrt{3}+2}$

③ $= \frac{(\sqrt{3}+2)^2 - 4\sqrt{3}}{\sqrt{3}(\sqrt{3}+2)}$

$= \frac{3 + 4\sqrt{3} + 4 - 4\sqrt{3}}{3 + 2\sqrt{3}}$

④ $= \frac{7}{3+2\sqrt{3}} \times \frac{3-2\sqrt{3}}{3-2\sqrt{3}}$

$= \frac{21 - 14\sqrt{3}}{3^2 - (2\sqrt{3})^2}$

$= \frac{21-14\sqrt{3}}{-3}$

$= \frac{14\sqrt{3}-21}{3}$

## Type B : Find the values of unknown constants by comparing surds

If $a + b\sqrt{n} = p + q\sqrt{n}$ , then $a = p$ and $b = q$ . Note : $a, b, p, q$ are rational numbers.

Eg : $a + b\sqrt{n} = 5 - 7\sqrt{n}$

$\Rightarrow a = 5 \ , \ b = -7$

Eg : $(x + y\sqrt{3})^2 = 19 + 8\sqrt{3}$

$x^2 + 2xy\sqrt{3} + 3y^2 = 19 + 8\sqrt{3}$

$\Rightarrow x^2 + 3y^2 = 19$

$\Rightarrow 2xy = 8$

Then solve this pair of simultaneous equations.

Eg : Given that $\frac{\sqrt{3}}{5+\sqrt{3}} - \frac{2}{2-\sqrt{3}} = a + b\sqrt{3}$ , where $a$ and $b$ are constants, find $a$ and $b$.

$\frac{\sqrt{3}}{5+\sqrt{3}} - \frac{2}{2-\sqrt{3}} = \frac{\sqrt{3}(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} - \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$ rationalise each fraction

$= \frac{5\sqrt{3}-3}{25-3} - \frac{2+\sqrt{3}}{4-3}$

$= \frac{5\sqrt{3}-3}{22} - \frac{2+\sqrt{3}}{1}$

$= \frac{5\sqrt{3} - 3 - 2 - \sqrt{3}}{22}$

$= \frac{-5+4\sqrt{3}}{22}$

$= -\frac{5}{22} + \frac{4}{22}\sqrt{3}$

$= -\frac{5}{22} + \frac{2}{11}\sqrt{3}$

$\therefore a = -\frac{5}{22} \ , \ b = \frac{2}{11}$

TIP

Check your answers by substituting back to the question and check if LHS = RHS.

## Type C: Solving surds $\sqrt{\phantom{x}} = \sqrt{\phantom{x}}$ and checking your answers

① Simplify all roots if possible.
② Rearrange and ensure each side of the equation only has one surd.
③ Square both sides and solve.
④ Check answers by substituting each $x$ value into the question's LHS and RHS.
⑤ Conclude the final answer(s).

Eg: $3+\sqrt{3x-5} = x$

② $\sqrt{3x-5} = x-3$

③ $3x-5 = (x-3)^2$

$3x-5 = x^2-6x+9$

$x^2-9x+14=0$

$(x-7)(x-2)=0$

$x=7$ or $x=2$

④ Check: When $x=7$,

LHS $= 3+\sqrt{3(7)-5}$
$= 7$
$=$ RHS

When $x=2$,

LHS $= 3+\sqrt{3(2)-5}$
$= 4$
$\neq$ RHS

⑤ $\therefore x=7$

Eg: $3\sqrt{2x-2} - \sqrt{4x+28} = 0$

① $3\sqrt{2x-2} - 2\sqrt{x+7} = 0$

② $3\sqrt{2x-2} = 2\sqrt{x+7}$

③ $\left(3\sqrt{2x-2}\right)^2 = \left(2\sqrt{x+7}\right)^2$

$9(2x-2) = 4(x+7)$

$18x-18 = 4x+28$

$14x = 46$

$x = \frac{23}{7}$

④ Check: LHS $= 3\sqrt{2\left(\frac{23}{7}\right)-2} - \sqrt{4\left(\frac{23}{7}\right)+28}$
$= 0$
$=$ RHS

⑤ $\therefore x = \frac{23}{7}$
$= 3\frac{2}{7}$

Eg: Squaring twice when solving

$\sqrt{x} = \sqrt{x-1}+3$

② $\sqrt{x}-3 = \sqrt{x-1}$

③ $\left(\sqrt{x}-3\right)^2 = \left(\sqrt{x-1}\right)^2$

② $x-6\sqrt{x}+9 = x-1$

$-6\sqrt{x} = -10$

$\sqrt{x} = \frac{5}{3}$

③ $x = \left(\frac{5}{3}\right)^2$
$= \frac{25}{9}$

④ Check: when $x = \frac{25}{9}$,

LHS $= \sqrt{\frac{25}{9}}$
$= \frac{5}{3}$

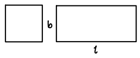
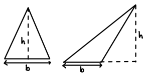
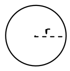
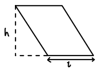
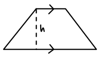
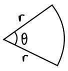
RHS $= \sqrt{\frac{25}{9}-1}+3$
$= \frac{13}{3}$
$\neq$ LHS

$\therefore x \neq \frac{25}{9}$

⑤ $\therefore$ There is no real solution.

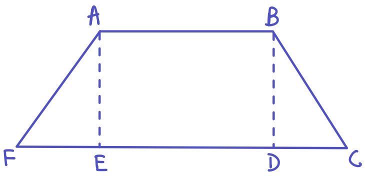
## Type D: Mensuration with surds (or real world application)

Quick recap on formulas (Sec 1-3 E-Math):

| 2D Figures | Shape | Perimeter | Area |
|---|---|---|---|
| Square / Rectangle |  | | length × breadth |
| Triangle |  | | $\frac{1}{2}$ × base × height |
| Circle |  | $2\pi r$ or $\pi d$ | $\pi r^2$ |
| Rhombus / Parallelogram |  | | length × height |
| Trapezium |  | | $\frac{1}{2}$ × (sum of parallel sides) × height |
| Sector |  | $\frac{\theta}{360°} \times \pi d + 2r$ | $\frac{\theta}{360°} \times \pi r^2$ |

| 3 D Figures | Surface Area | Volume |
|---|---|---|
| Cube / Cuboid | Add 6 sides | l × b × h |
| Cylinder | $\pi dh + 2\pi r^2$ | $\pi r^2 h$ |
| Prism | perimeter of cross section $\times$ *height* + 2 × Area of cross section | area of cross section × height |
| Cone | $\pi r l + \pi r^2$ | $\frac{1}{3} \times \pi r^2 \times$ height |
| Pyramid | add all sides area | $\frac{1}{3}$ × base area × height |
| Sphere | $4\pi r^2$ | $\frac{4}{3}\pi r^3$ |

Eg: The trapezium ABCF, where AB is parallel to CF, has an area of $\frac{1}{2}(21\sqrt{3} - 30)$ cm².




Given that the length of AB is $(5+2\sqrt{3})$ cm and the length of FC is $(10\sqrt{3}-2)$ cm and also that CD = EF, find the height of the trapezium in the form $a + \sqrt{b}$ , where a and b are integers.

[Source: Crescent Girls' School/2024/WA2/Q3]

$$\frac{1}{2} \times (5+2\sqrt{3} + 10\sqrt{3} - 2) \times \text{height} = \frac{1}{2}(21\sqrt{3} - 30)$$

$$\text{Height} = \frac{21\sqrt{3} - 30}{12\sqrt{3} + 3} \quad \div 3 \text{ for every term}$$

$$= \frac{7\sqrt{3} - 10}{4\sqrt{3} + 1} \times \frac{4\sqrt{3} - 1}{4\sqrt{3} - 1} \quad \text{rationalise}$$

$$= \frac{(7\sqrt{3} - 10)(4\sqrt{3} - 1)}{(4\sqrt{3})^2 - 1^2}$$

$$= \frac{28(3) - 7\sqrt{3} - 40\sqrt{3} + 10}{48 - 1}$$

$$= \frac{94 - 47\sqrt{3}}{47}$$

$$= (2 - \sqrt{3})\text{ cm}$$

An open cuboid bin has a square base of side $(3+\sqrt{5})$ m. The capacity of the bin is $(58+26\sqrt{5})$ m³.

Find the height of the bin in the form $(a+b\sqrt{5})$ m, where $a$ and $b$ are integers.

[Source: Nan Hua High School/2024/WA2/Q4]

$$\text{Base area} = (3+\sqrt{5})^2$$

$$= 9 + 6\sqrt{5} + 5$$

$$= 14 + 6\sqrt{5}$$

$$\text{Height} = \frac{\text{Volume}}{\text{Base area}}$$

$$= \frac{58+26\sqrt{5}}{14+6\sqrt{5}} \times \frac{14-6\sqrt{5}}{14-6\sqrt{5}} \quad \text{rationalise}$$

$$= \frac{(58+26\sqrt{5})(14-6\sqrt{5})}{14^2 - (6\sqrt{5})^2}$$

$$= \frac{812 - 348\sqrt{5} + 364\sqrt{5} - 780}{196 - 180}$$

$$= \frac{32 + 16\sqrt{5}}{16}$$

$$= (2 + \sqrt{5})\text{ m}$$