CHAPTER 2 Numbers

Note: Expressions such as $\frac{8}{0}$, where the denominator is zero, are undefined.

- Real Numbers
  - Rational Numbers
    - Integers
      - Whole Numbers
        - Zero, 0
        - Positive Integers
          - One, 1
          - Prime Numbers e.g. 2, 3, 5
          - Composite Numbers e.g. 4, 6, 8
      - Negative Integers e.g. -5, -101
    - Fractions e.g. $\frac{1}{7}$, $-1\frac{1}{5}$
    - Decimals
      - Recurring Decimals e.g. $0.\dot{3}$, $1.\dot{5}\dot{6}$, $\frac{1}{7}$, $-\frac{284}{11}$
      - Terminating Decimals e.g. 1.5, $-1\frac{1}{5}$, 3, $\frac{9}{4}$
  - Irrational Numbers
    - Non-terminating and non-recurring decimals e.g. π, $\sqrt[3]{7}$, $\frac{1}{\sqrt{3}}$, $9^{\frac{1}{3}}$, $\frac{\sqrt{2}}{3}$
- imaginary numbers (not a real number) e.g. $\sqrt{-8}$, $\sqrt{-103}$

TIP

Key the number into the calculator and press "=" → If the answer is a fraction, the number is rational.

Calculator Hack!

To convert a recurring decimal to a fraction, type the repeating part many times and press "=".

e.g. $1.1\dot{3}$ → Type 1.1333333333333 = $\frac{17}{15}$

Definitions	
Real numbers	Numbers that can be represented as a point on a number line.
Rational numbers	Numbers that can be expressed in the form of $\frac{a}{b}$ where $b \neq 0$
Irrational numbers	Numbers that cannot be expressed in the form of $\frac{a}{b}$.
Terminating decimals	Decimals will stop / end after a few digits.
Recurring decimals	Decimals that never end and have one or more digits that repeat forever.
Non-terminating and non-recurring decimals	Decimals that never end, do not repeat and continue forever.
Prime numbers	Whole numbers that have exactly two different factors, 1 and itself.
Composite numbers	Whole numbers that have more than two different factors.

- **Arranging numbers** (small to big ↑)

E.g. Arrange the numbers in ascending order.

$-\pi$, $1\frac{3}{7}$, $(-1.6)^2$, $\sqrt{2}$, $1.4\dot{1}$

$-\pi = -3.1415$

$1\frac{3}{7} = 1.4285$

$(-1.6)^2 = 2.5600$

$\sqrt{2} = 1.4142$

$1.4\dot{1} = 1.4111$

Ans: $-\pi$, $1.4\dot{1}$, $\sqrt{2}$, $1\frac{3}{7}$, $(-1.6)^2$

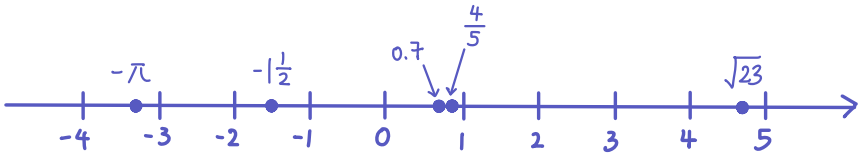
TIP

Use calculator to convert all numbers to decimal. Compare the decimals and put original numbers as your final answer.

- **Number line** (use dots or cross to mark the numbers)

E.g. Represent the numbers on the number line.

0.7, $-1\frac{1}{2}$, $-\pi$, $\sqrt{23}$, $\frac{4}{5}$



- **Multiply / Divide negative numbers**

1st sign	2nd sign	Result
+	+	+
+	−	−
−	+	−
−	−	+

- **Inequality signs**

Symbol	Meaning	Example
$<$	less than	$2 < 6$ $x < 7$
$\leqslant$	less than or equal to	$x \leqslant 10$
$>$	more than	$8 > 1$ $x > 12$
$\geqslant$	more than or equal to	$x \geqslant 5$

- **Order of operation (BODMAS)**

1st to tackle: Brackets

↓

Orders (powers and square roots)

↓

Division / Multiplication (from left to right)

↓

Addition / Subtraction (from left to right)

Eg 1: $(-3)^2 \times 3 - 2[5 \times (-2)]$

$= 9 \times 3 - 2[5 \times (-2)]$

$= 9 \times 3 - 2(-10)$

$= 27 \quad -2(-10)$

$= 27 \quad +20$

$= 47$

TIP

Always check your answer by keying the question exactly into your calculator.

Eg 2: $(-4)^3 - 72 \div [-6 - \sqrt{64} + 2]$

$= -64 - 72 \div (-6 - 8 + 2)$

$= -64 - 72 \div (-12)$

$= -64 + 6$

$= -58$

Eg 3: $[15 + (-4)^2] + 3 \times (2-5)^2 \div (-3)$

$= [15 + 16] + 3 \times (-3)^2 \div (-3)$

$= 31 + 3 \times 9 \div (-3)$

$= 31 + 27 \div (-3)$

$= 31 - 9$

$= 22$

- Order of operation with fractions involved
  - Change mixed numbers to improper fractions when solving.
  - Leave final answer in proper / mixed numbers and in the simplest form.

a) Add / Subtract fractions $\longrightarrow$ Make denominators the same then combine fractions.

Eg: $\frac{4}{5} + 2\frac{1}{2} = \frac{4}{5} + \frac{5}{2} = \frac{8}{10} + \frac{25}{10} = \frac{33}{10} = 3\frac{3}{10}$

b) Multiply fractions $\longrightarrow$ Numerator $\times$ Numerator, denominator $\times$ denominator

Eg: $\frac{4}{5} \times 2\frac{1}{2} = \frac{4}{5} \times \frac{5}{2} = \frac{4}{2} = 2$

c) Divide fractions $\longrightarrow$ Keep, change, flip and multiply the fractions.

Eg: $\frac{4}{5} \div 2\frac{1}{2} = \frac{4}{5} \div \frac{5}{2} = \frac{4}{5} \times \frac{2}{5} = \frac{8}{25}$

d) Square or square root fractions $\longrightarrow$ square / square root <u>both</u> the numerator and denominator

Eg: $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$ $\quad \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$ $\quad \left(-\frac{2}{3}\right)^3 = -\frac{8}{27}$

$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$ $\quad \sqrt[3]{-\frac{27}{64}} = \frac{\sqrt[3]{-27}}{\sqrt[3]{64}} = \frac{-3}{4} = -\frac{3}{4}$

Eg 1: $(-2)^2 + \left(5 - \frac{1}{2} \times 7 - \frac{2}{3}\right)$

$= 4 + \left(5 - \frac{7}{2} - \frac{2}{3}\right)$

$= 4 + \left(\frac{30}{6} - \frac{12}{6} - \frac{4}{6}\right)$

$= 4 + \frac{14}{6}$

$= \frac{24}{6} + \frac{14}{6}$

$= \frac{38}{6}$

$= 6\frac{1}{3}$

Eg 2: $2\frac{1}{6} - \left(-7\frac{1}{3}\right) - \left(-\frac{3}{4}\right)^2$

$= \frac{13}{6} - \left(-7\frac{1}{3}\right) - \frac{9}{16}$

$= \frac{13}{6} + \frac{22}{3} - \frac{9}{16}$

$= \frac{13}{6} + \frac{44}{6} - \frac{9}{16}$

$= \frac{57}{6} - \frac{9}{16}$

$= \frac{456}{48} - \frac{27}{48}$

$= \frac{429}{48}$

$= 8\frac{15}{16}$

Commonly tested Word Problems

Eg 1: The altitudes of some places are as follows:
[Source: Presbyterian High School / 2023 / WA1 / Q2]

Location	Altitudes
Dead Sea	-430 m
Lake Assal	- 153 m
Mexico City	2240 m
Mount Everest	8848 m
Westmorland	-48 m

(a) List the places above sea level.

Mexico City and Mount Everest.

(b) Calculate the difference in altitude between Lake Assal and Dead Sea.

Difference = -153 - (-430) use bigger number − smaller number
= 277 m

Eg 2: In March, the minimum temperature is -19°C while the maximum temperature is 28°C.

(a) Find the difference between the minimum and maximum temperature.

Difference = 28 - (-19)
= 47°C

(b) In April, the minimum temperature is 24°C colder than the mininimum temperature in March. Find the minimum temperature in April.

Minimum temperature in April = -19 - 24
= -43°C