CHAPTER 3 AM Surds

Note: No mixed number with surds.
✓ $\frac{5}{3}\sqrt{x}$ ✗ $1\frac{2}{3}\sqrt{x}$

1)

	Rules	Examples
Multiplication	$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $x\sqrt{a} \times y\sqrt{b} = xy\sqrt{ab}$ Number × Number, Surd × Surd	$\sqrt{5} \times \sqrt{3} = \sqrt{15}$ $6\sqrt{5} \times 8\sqrt{3} = 48\sqrt{15}$
Multiplying Same surds	$\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$ $\sqrt{a} \times \sqrt{a} \times \sqrt{a} = (\sqrt{a})^3$ $b\sqrt{a} \times b\sqrt{a} = (b\sqrt{a})^2 = b^2a$	$\sqrt{5} \times \sqrt{5} = 5$ $\sqrt{5} \times \sqrt{5} \times \sqrt{5} = 5\sqrt{5}$ $3\sqrt{5} \times 3\sqrt{5} = 9(5) = 45$
Division	$\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	$\sqrt{10} \div \sqrt{2} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$
Addition	$a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$	$8\sqrt{5} + 3\sqrt{5} = 11\sqrt{5}$
Subtraction	$a\sqrt{x} - b\sqrt{x} = (a-b)\sqrt{x}$	$8\sqrt{5} - 3\sqrt{5} = 5\sqrt{5}$

2) Rationalising

- "Get rid" of surds in the denominator
- Apply special identity: $(a+b)(a-b) = a^2 - b^2$

Eg: Rationalise the following

$$\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

$$\frac{5}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{6(3)} = \frac{5\sqrt{3}}{18}$$

$$\frac{5}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{5(\sqrt{3}-2)}{(\sqrt{3})^2 - 2^2} = \frac{5\sqrt{3}-10}{3-4} = 10 - 5\sqrt{3}$$

$$\frac{7}{2-5\sqrt{3}} \times \frac{2+5\sqrt{3}}{2+5\sqrt{3}} = \frac{7(2+5\sqrt{3})}{} = \frac{14+35\sqrt{3}}{4-75} = -\frac{14+35\sqrt{3}}{71}$$

3) Simplifying surds

- Use perfect squares: 4, 9, 16, 25, 36, 49 ...

Eg: $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$\sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$

$2\sqrt{50} = 2\sqrt{25 \times 2} = 2 \times 5\sqrt{2} = 10\sqrt{2}$

TIP

To check if you simplified correctly, type the question and your final answer in the calculator and check if the values match.

Types of Surds Questions and Steps to tackle them :

Type A : Simplify and / or rationalise expressions involving surds

① Simplify all surds to the simplest form (eg : $\sqrt{32} = 4\sqrt{2}$)

Tip : Pick the smallest root that you see in the question and change all to that root.

② Combine all the like terms (those with same root)

③ Combine fractions into one single fraction by making same denominator.

④ For fractions, check denominator : If there are surds, rationalise.

Eg : Simplify $\sqrt{27} + 2\sqrt{48} - \frac{10}{\sqrt{3}}$

$\sqrt{27} + 2\sqrt{48} - \frac{10}{\sqrt{3}}$

① $= 3\sqrt{3} + 2\times 4\sqrt{3} - \frac{10}{\sqrt{3}}$

② $= 3\sqrt{3} + 8\sqrt{3} - \frac{10}{\sqrt{3}}$

④ $= 11\sqrt{3} - \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ← we can rationalise for just one specific term when needed

TIP

$= 11\sqrt{3} - \frac{10\sqrt{3}}{3}$

$= 11\sqrt{3} - \frac{10}{3}\sqrt{3}$

$= \frac{23}{3}\sqrt{3}$

Eg : Simplify $\frac{\sqrt{3}+2}{\sqrt{3}} - \frac{4}{\sqrt{3}+2}$

③ $= \frac{(\sqrt{3}+2)^2 - 4\sqrt{3}}{\sqrt{3}(\sqrt{3}+2)}$

$= \frac{3 + 4\sqrt{3} + 4 - 4\sqrt{3}}{3 + 2\sqrt{3}}$

④ $= \frac{7}{3 + 2\sqrt{3}} \times \frac{3 - 2\sqrt{3}}{3 - 2\sqrt{3}}$

$= \frac{21 - 14\sqrt{3}}{3^2 - (2\sqrt{3})^2}$

$= \frac{21 - 14\sqrt{3}}{-3}$

$= \frac{14\sqrt{3} - 21}{3}$

Type B : Find the values of unknown constants by comparing surds

If $a + b\sqrt{n} = p + q\sqrt{n}$, then $a = p$ and $b = q$. Note : a, b, p, q are rational numbers.

Eg : $a + b\sqrt{n} = 5 - 7\sqrt{n}$

$\Rightarrow a = 5 \;,\; b = -7$

Eg : $(x + y\sqrt{3})^2 = 19 + 8\sqrt{3}$

$x^2 + 2xy\sqrt{3} + 3y^2 = 19 + 8\sqrt{3}$

$\Rightarrow x^2 + 3y^2 = 19$

$\Rightarrow 2xy = 8$

Then solve this pair of simultaneous equations.

Eg : Given that $\frac{\sqrt{3}}{5+\sqrt{3}} - \frac{1}{1-\sqrt{3}} = a + b\sqrt{3}$, where a and b are constants, find a and b.

$\frac{\sqrt{3}}{5+\sqrt{3}} - \frac{1}{2-\sqrt{3}} = \frac{\sqrt{3}(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} - \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$ rationalise each fraction

$= \frac{5\sqrt{3} - 3}{25 - 3} - \frac{2+\sqrt{3}}{4 - 3}$

$= \frac{5\sqrt{3} - 3}{22} - \frac{2+\sqrt{3}}{1}$

$= \frac{5\sqrt{3} - 3 - 2 - \sqrt{3}}{22}$

$= \frac{-5 + 4\sqrt{3}}{22}$

$= -\frac{5}{22} + \frac{4}{22}\sqrt{3}$

$= -\frac{5}{22} + \frac{2}{11}\sqrt{3}$

$\therefore a = -\frac{5}{22} \;,\; b = \frac{2}{11}$

TIP

Check your answers by substituting back to the question and check if LHS = RHS.

Type C: Solving surds $\sqrt{\quad} = \sqrt{\quad}$ and checking your answers

① Simplify all roots if possible.
② Rearrange and ensure each side of the equation only has one surd.
③ Square both sides and solve.
④ Check answers by substituting each x value into the question's LHS and RHS.
⑤ Conclude the final answer(s).

Eg: $3 + \sqrt{3x-5} = x$

② $\sqrt{3x-5} = x-3$

③ $3x-5 = (x-3)^2$

$3x-5 = x^2-6x+9$

$x^2-9x+14=0$

$(x-7)(x-2)=0$

$x=7$ or $x=2$

④ Check: When $x=7$,

LHS $= 3+\sqrt{3(7)-5}$
$= 7$
$=$ RHS

When $x=2$,

LHS $= 3+\sqrt{3(2)-5}$
$= 4$
$\neq$ RHS

⑤ $\therefore x = 7$

Eg: $3\sqrt{2x-2} - \sqrt{4x+28} = 0$

① $3\sqrt{2x-2} - 2\sqrt{x+7} = 0$

② $3\sqrt{2x-2} = 2\sqrt{x+7}$

③ $\left(3\sqrt{2x-2}\right)^2 = \left(2\sqrt{x+7}\right)^2$

$9(2x-2) = 4(x+7)$

$18x-18 = 4x+28$

$14x = 46$

$x = \frac{23}{7}$

④ Check: LHS $= 3\sqrt{2\left(\frac{23}{7}\right)-2} - \sqrt{4\left(\frac{23}{7}\right)+28}$
$= 0$
$=$ RHS

⑤ $\therefore x = \frac{23}{7}$
$= 3\frac{2}{7}$

Eg: Squaring twice when solving

$\sqrt{x} = \sqrt{x-1} + 3$

② $\sqrt{x} - 3 = \sqrt{x-1}$

③ $\left(\sqrt{x}-3\right)^2 = \left(\sqrt{x-1}\right)^2$

② $x - 6\sqrt{x} + 9 = x-1$

$-6\sqrt{x} = -10$

$\sqrt{x} = \frac{5}{3}$

③ $x = \left(\frac{5}{3}\right)^2$
$= \frac{25}{9}$

④ Check: when $x = \frac{25}{9}$,

LHS $= \sqrt{\frac{25}{9}}$
$= \frac{5}{3}$

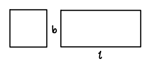
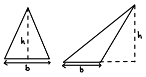
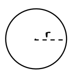
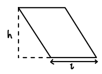
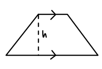
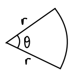
RHS $= \sqrt{\frac{25}{9}-1} + 3$
$= \frac{13}{3}$

$\therefore x \neq \frac{25}{9}$

⑤ $\therefore$ There is no real solution.

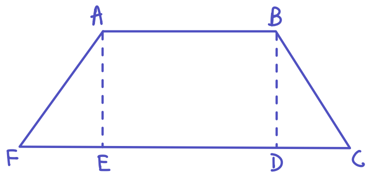
Type D: Mensuration with surds (or real world application)

Quick recap on formulas (Sec 1-3 E-Math):

2D Figures	Shape	Perimeter	Area
Square / Rectangle			length × breadth
Triangle			$\frac{1}{2}$ × base × height
Circle		$2\pi r$ or πd	πr^2
Rhombus / Parallelogram			length × height
Trapezium			$\frac{1}{2}$ × (sum of parallel sides) × height
Sector		$\frac{\theta}{360°} \times \pi d + 2r$	$\frac{\theta}{360°} \times \pi r^2$

3 D Figures	Surface Area	Volume
Cube / Cuboid	Add 6 sides	l × b × h
Cylinder	$\pi dh + 2\pi r^2$	$\pi r^2 h$
Prism	perimeter of cross section $\times$ *height* + 2 × Area of cross section	area of cross section × height
Cone	$\pi r l + \pi r^2$	$\frac{1}{3} \times \pi r^2 \times$ height
Pyramid	add all sides area	$\frac{1}{3}$ × base area × height
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Eg: The trapezium ABCF, where AB is parallel to CF, has an area of $\frac{1}{2}(21\sqrt{3} - 30)$ cm².



Given that the length of AB is $(5+2\sqrt{3})$ cm and the length of FC is $(10\sqrt{3}-2)$ cm and also that CD = EF, find the height of the trapezium in the form $a + \sqrt{b}$, where a and b are integers.

[Source: Crescent Girls' School/2024/WA2/Q3]

$$\frac{1}{2} \times (5+2\sqrt{3} + 10\sqrt{3} - 2) \times \text{height} = \frac{1}{2}(21\sqrt{3} - 30)$$

$$\text{Height} = \frac{21\sqrt{3} - 30}{12\sqrt{3} + 3}$$ ÷3 for every term

$$= \frac{7\sqrt{3} - 10}{4\sqrt{3} + 1} \times \frac{4\sqrt{3} - 1}{4\sqrt{3} - 1}$$ rationalise

$$= \frac{(7\sqrt{3} - 10)(4\sqrt{3} - 1)}{(4\sqrt{3})^2 - 1^2}$$

$$= \frac{28(3) - 7\sqrt{3} - 40\sqrt{3} + 10}{48 - 1}$$

$$= \frac{94 - 47\sqrt{3}}{47}$$

$$= (2 - \sqrt{3})\text{ cm}$$

An open cuboid bin has a square base of side $(3+\sqrt{5})$ m. The capacity of the bin is $(58+26\sqrt{5})$ m³.

Find the height of the bin in the form $(a+b\sqrt{5})$ m, where a and b are integers.

[Source: Nan Hua High School/2024/WA2/Q4]

$$\text{Base area} = (3+\sqrt{5})^2$$

$$= 9 + 6\sqrt{5} + 5$$

$$= 14 + 6\sqrt{5}$$

$$\text{Height} = \frac{\text{Volume}}{\text{Base area}}$$

$$= \frac{58 + 26\sqrt{5}}{14 + 6\sqrt{5}} \times \frac{14 - 6\sqrt{5}}{14 - 6\sqrt{5}}$$ rationalise

$$= \frac{(58 + 26\sqrt{5})(14 - 6\sqrt{5})}{14^2 - (6\sqrt{5})^2}$$

$$= \frac{812 - 348\sqrt{5} + 364\sqrt{5} - 780}{196 - 180}$$

$$= \frac{32 - 16\sqrt{5}}{16}$$

$$= (2 - \sqrt{5})\text{ m}$$