CHAPTER 2 Linear Inequalities

1) Inequality Signs

$>$	greater than
$<$	less than
$\geq$	greater than or equal to
$\leq$	less than or equal to

2) When to flip the inequality sign

a) Do not flip when you do the following on both sides of an inequality

Add a number	Subtract a number
Eg: $x > y$, $x+3 > y+3$	Eg: $a \leq b$, $a-8 \leq b-8$
Multiply by a positive number	**Divide by a positive number**
Eg: $x \geq y$, $3x \geq 3y$	Eg: $a < b$, $\frac{a}{2} < \frac{b}{2}$

b) Flip when you do the following on both sides of an inequality

Multiply by a negative number	Divide by a negative number
Eg: $x > y$, $-2x < -2y$	Eg: $a \leq b$, $\frac{a}{-5} \geq \frac{b}{-5}$
Swap the position of left-hand side and right-hand side	
Eg: $\frac{1}{3} \geq x$, $x \leq \frac{1}{3}$	

2) Solving linear inequality

- Solve means to find the range of values of x that make the inequality true.
- Method: ✓ Make one x alone on the left-hand side
  ✓ Leave final answer in mixed number (for fractions)

a) Without fractions

Eg: $-7y + 8 \leq -2(y-6)$

$-7y + 8 \leq -2y + 12$

$-7y + 2y \leq 12 - 8$

$-5y \leq 4$ ↘ divide by -5, flip the sign

This is the solution, to be written on the answer line. → $\boxed{y \geq -\frac{4}{5}}$

Eg: $18 > 6f + 10(f-1)$

$18 > 6f + 10f - 10$

$18 - 10 > 16f$

$8 > 16f$ ↘ swap sides, flip sign

$16f < 8$

$f < \frac{8}{16}$

$\boxed{f < \frac{1}{2}}$

b) With fractions

(i) No cross multiply

Eg: $\frac{x}{2} > -\frac{1}{6}$

$\div \frac{1}{2}$ ↘ $\frac{1}{2}x > -\frac{1}{6}$ ↘ $\div \frac{1}{2}$

$x > -\frac{1}{3}$

(ii) Cross multiply (Top left stays left!)

$\frac{x}{2} > \frac{-1}{6}$ ← The denominators are positive, so the inequality sign does not flip when we cross multiply

$6x > -2$

$x > -\frac{2}{6}$

$x > -\frac{1}{3}$

$\frac{x}{2} > \frac{1}{-6}$ ← The negative sign at the denominator will cause the inequality sign to flip when we cross multiply.

$-6x < 2$ ↘ $\div(-6)$ (divide by -6, the sign flips again)

$x > -\frac{2}{6}$

$x > -\frac{1}{3}$

Eg: (a) $4 - \frac{3}{4}(x-3) > -\frac{1}{2}$

$4 - \frac{3}{4}x + \frac{9}{4} > -\frac{1}{2}$

$-\frac{3}{4}x > -\frac{1}{2} - \frac{9}{4} - 4$

$\div(-\frac{3}{4})$ ↘ $-\frac{3}{4}x > -\frac{27}{4}$ ↘ divide by $-\frac{3}{4}$, flip the sign

$x < 9$

(b) Hence, write down the largest value of x if x is a perfect cube.

Largest perfect cube = 8

Eg: (a) $\frac{2(3m-7)}{-5} \leq 9$ ↘ cross multiply and flip sign

$2(3m-7) \geq -45$

$6m - 14 \geq -45$

$6m \geq -31$

$m \geq -\frac{31}{6}$

$m \geq -5\frac{1}{6}$

(b) Using your solution in (a), find the smallest integer value of m.

Smallest integer = -5

3) Representing solution on a number line

use	o	for	<	and	>
use	•	for	≤	and	≥

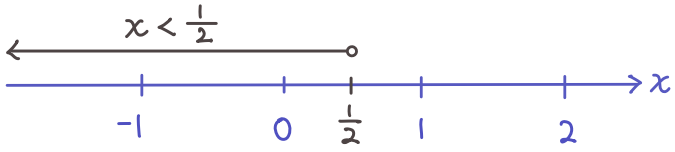
Solve the following inequalities and represent each solution on a number line.

Eg 1: $-2x + 8 < 7$

$-2x < 7 - 8$

$-2x > -1$ (divide by -2, flip the sign)

This is the solution, to be written on the answer line. → $x < \frac{1}{2}$



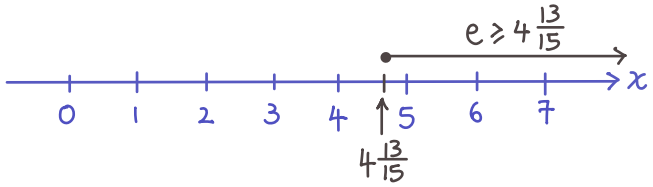
Eg 2: $3(e - 8) \geq -2e + \frac{1}{3}$

$3e - 24 \geq -2e + \frac{1}{3}$

$3e + 2e \geq \frac{1}{3} + 24$

$5e \geq \frac{73}{3}$

$e \geq 4\frac{13}{15}$



4) Word problem involving inequality

- Always define the variable if it is not given.
  Eg: Let the amount of money Sarah has be $\$x$.
- Identify keywords to determine the inequality sign.
- Check that your answer makes sense and fits the context of the question.
  Eg: Number of items must be a whole number

Common Keywords :

At most / budget	≤
At least	≥

Eg: A coach bought some basketballs at \$18 each and some soccer balls at \$16 each. He bought a total of 20 balls and the cost of basketballs was at least \$40 more than the cost of soccer balls. Form an inequality and find the minimum number of basketballs bought.

Let the number of basketballs be x.

Number of soccer balls $= 20 - x$

$18x - 16(20 - x) \geq 40$

$18x - 320 + 16x \geq 40$

$18x + 16x \geq 40 + 320$

$34x \geq 360$

$x \geq 10\frac{10}{17}$

$\therefore$ Minimum number of basketballs bought $= 11$