

CHAPTER 3

EM

Indices & Standard Form

Compound interest

Singular: index
Plural: indices
("power")

Eg: 7³
base

	Laws of Indices	Examples
Same base laws	1) $a^m \times a^n = a^{m+n}$	$2^3 \times 2^{-7} = 2^{-4}$
	2) $a^m \div a^n = a^{m-n} = \frac{a^m}{a^n}$	$\frac{4^{-2}}{4^{-8}} = 4^{-2} \div 4^{-8} = 4^{-2-(-8)} = 4^6$
	3) $(a^m)^n = a^{mn}$	$(5^{\frac{1}{3}})^{-2} = 5^{-\frac{2}{3}}$
Same power laws	4) $a^n \times b^n = (ab)^n = a^n b^n$	$(-3)^4 \times (-\frac{5}{3})^4 = 5^4$
	5) $a^n \div b^n = (\frac{a}{b})^n = \frac{a^n}{b^n}$	$\frac{27^{-3}}{9^{-3}} = 3^{-3}$
	6) $a^0 = 1$	$(10^{-2} \times 5^{-3} \times \frac{1}{2})^0 = 1$
	7) $a^{-n} = \frac{1}{a^n}$	$5^{-3} = \frac{1}{5^3}$ $\frac{5a^{-3}}{1} = \frac{5}{a^3}$ $\frac{(5a)^{-3}}{1} = \frac{1}{(5a)^3} = \frac{1}{125a^3}$
8)	$(\frac{a}{b})^{-n} = (\frac{b}{a})^n$ <small>same as</small> $\frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$ <small>same as</small>	$(\frac{7^{-2}a^3}{b^5})^{-4} = (\frac{b^5}{7^{-2}a^3})^4$ $= \frac{b^{20}}{7^{-8}a^{12}}$ $= \frac{7^8 b^{20}}{a^{12}}$
9)	$\sqrt[n]{a} = a^{\frac{1}{n}}$	<small>empty = 2</small> $\sqrt[2]{9a} = 3a^{\frac{1}{2}}$ $\sqrt[5]{a^2c} = (a^2c)^{\frac{1}{5}} = a^{\frac{2}{5}}c^{\frac{1}{5}}$
10)	$\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$	$\sqrt[7]{(a^{-2}b^5)^3} = (a^{-2}b^5)^{\frac{3}{7}} = a^{-\frac{6}{7}}b^{\frac{15}{7}} = \frac{b^{\frac{15}{7}}}{a^{\frac{6}{7}}}$

The power '-3' belongs to 'a' only. So only 'a' will switch to the bottom and change to positive index.

eg. Simplify the following, express your answer in positive index notation.

$$\begin{aligned}
 (a) \sqrt[3]{x^3 y^{-2}} &= (x^3 y^{-2})^{\frac{1}{3}} \\
 &= \left(\frac{x^3}{y^2}\right)^{\frac{1}{3}} \\
 &= \frac{x}{y^{\frac{2}{3}}}
 \end{aligned}$$

TIPS

- ✓ Remove roots
- ✓ Remove negative
- ✓ Remove bracket
- ✓ Combine fractions
- * If there's \div , need to Keep, Change, Flip
- ✓ Simplify

$$\begin{aligned}
 (b) \quad & \left(\frac{b^{-4}}{ac^{-3}}\right)^3 \div \left(\frac{\sqrt{a}}{c^3}\right)^{-2} \\
 & = \left(\frac{b^{-4}}{ac^{-3}}\right)^3 \div \left(\frac{a^{\frac{1}{2}}}{c^3}\right)^{-2} \quad \leftarrow \text{Remove roots} \\
 & = \left(\frac{c^3}{ab^4}\right)^3 \div \left(\frac{c^3}{a^{\frac{1}{2}}}\right)^2 \quad \leftarrow \text{Remove negative} \\
 & = \frac{c^9}{a^3b^{12}} \div \frac{c^6}{a} \quad \leftarrow \text{Remove bracket} \\
 & = \frac{c^9}{a^3b^{12}} \times \frac{a}{c^6} \quad \leftarrow \text{Change } \div \text{ to } \times \\
 & = \frac{ac^9}{a^3b^{12}c^6} \quad \leftarrow \text{Combine into one single fraction} \\
 & = \frac{c^3}{a^2b^{12}} \quad \leftarrow \text{Simplify same base or same index}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & \frac{-a^2}{27ab^0} \div \frac{12b^{-3}}{\sqrt{81a^4b^3}} \\
 & = \frac{-a^2}{27a} \div \frac{12b^{-3}}{9a^2b^{\frac{3}{2}}} \\
 & = \frac{-a^2}{27a} \div \frac{12}{9a^2b^{\frac{3}{2}+3}} \\
 & = \frac{-a}{27} \times \frac{9a^2b^{\frac{9}{2}}}{12} \\
 & = \frac{-9a^3b^{\frac{9}{2}}}{324} \\
 & = -\frac{a^3b^{\frac{9}{2}}}{36}
 \end{aligned}$$

2) Solving equations

- "Solve" means to find the value of x .
- Method: Convert to the same base and compare the powers

$$\begin{array}{|l}
 a^x = a^n \\
 \Rightarrow x = n
 \end{array}
 \quad \text{where } a \neq -1, 0 \text{ or } 1$$

Type 1: Change to same base directly

$$\begin{aligned}
 \text{eg. } & 16^{1-x} = 8(2^{2x}) \\
 & (2^4)^{1-x} = (2^3)(2^{2x}) \\
 & 2^{4-4x} = 2^{3+2x} \\
 \Rightarrow & 4-4x = 3+2x \\
 & -6x = -1 \\
 & x = \frac{1}{6}
 \end{aligned}$$

Type 2: Same powers, combine the bases first

$$\begin{aligned}
 \text{eg. } & 3^{x+5} \times 2^{x+5} = \frac{1}{36} \\
 & 6^{x+5} = 36^{-1} \\
 & 6^{x+5} = 6^{-2} \\
 \Rightarrow & x+5 = -2 \\
 & x = -7
 \end{aligned}$$

TIP Check your answer by substituting your solution to the LHS and RHS of the question separately.

c) Factorise, change base, combine base, Compare powers.

- Factorise when you see addition / subtraction of same / common terms.

eg. Solve $8^2 + 8^2 + 8^2 + 8^2 = 8^n$

$$8^2 (1+1+1+1) = 8^n$$

$$8^2 (4) = 8^n$$

$$2^6 (2^2) = 2^{3n}$$

$$2^8 = 2^{3n}$$

$$\Rightarrow 8 = 3n$$

$$n = \frac{8}{3}$$

$$= 2\frac{2}{3}$$

eg. Show that $2^{15} + 2^{13}$ is divisible by 5.

$$2^{15} + 2^{13} = 2^{13} (2^2 + 1)$$

$$= 2^{13} (5)$$

Since $2^{15} + 2^{13}$ has a factor of 5,
it is divisible by 5.

3) Other types of questions involving indices

eg. Express $81^4 \times 9^{2p}$ as a power of 3.

$$(3^4)^4 \times (3^2)^{2p} = 3^{16} \times 3^{4p}$$

$$= 3^{16+4p}$$

Means that the
base is 3.

eg. (a) Solve $3x^2 - 28x + 9 = 0$

$$(3x-1)(x-9) = 0$$

$$3x-1 = 0 \quad \text{or} \quad x-9 = 0$$

$$x = \frac{1}{3}$$

$$x = 9$$

(b) Hence, solve the equation $3(3^{2x}) - 28(3^x) + 9 = 0$

$$3(3^x)^2 - 28(3^x) + 9 = 0$$

From (a), $3^x = \frac{1}{3}$ or $3^x = 9$

$$3^x = 3^{-1}$$

$$3^x = 3^2$$

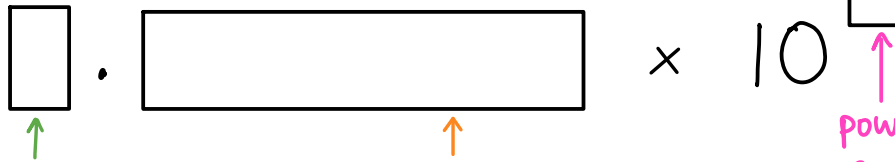
$$\Rightarrow x = -1$$

$$x = 2$$

The 'x' in
part (a) has
been replaced
by '3^x'.

4) Standard form

✓ Format.



- can only have 1 digit
- $0 < \text{digit} < 10$

Remaining digits

power must be an integer

Not in standard form	Standard form
6 789 000	6.789×10^6
0.0000000000358	3.58×10^{-11}
0.284×10^3	2.84×10^2
10×10^{-8}	1.0×10^{-7}
800×10^{-1}	8×10^1

Calculate and give your answers in standard form, correct to 3 significant figures.

Examples	Standard form
3581×0.15	$= 5.37 \times 10^2$
$45 \times 10^{-2} \times 5.874 \times 10^8$	$= 2.64 \times 10^8$
$6.8 \times 10^{-7} - 3.79 \times 10^5$	$= -3.79 \times 10^5$
4×2189	$= 8.76 \times 10^3$
$(92.108 \times 10^3 + 4.8 \times 10^{-7}) \times (3.72 \times 10^4)$	$= 3.43 \times 10^9$
$9.6 \times 10^5 - 3.2 \times 10^5 = (9.6 - 3.2) \times 10^5$	$= 6.4 \times 10^5$
$\frac{3.111 \times 10^7 \div (4.29 \times 10^{-3} - 3 \times 10^6)}{-5.5 \times 10^4 - 2 \times 10^{-1}}$	$= 1.89 \times 10^{-4}$
$\frac{\sqrt{\frac{3}{4} \times 841 + (3^2 - 4)} + 90.81^3}{(2.5 \times 10^{-2} - 8) + 33} + \sqrt[3]{\frac{181}{197}}$	$= 2.99 \times 10^4$

✓ use calculator to check



Shift → menu → 3: Number format → 2: Sci → 3

FYI: To choose the significant figure



How to set calculator back to normal mode:

Shift → menu → 3: Number format → 3: Norm → 2

✓ SI Prefix

power of 10	SI Prefix	Symbol	English word
10^{12}	tera-	T	trillion
10^9	giga-	G	billion
10^6	mega-	M	million
10^3	Kilo-	k	thousand
10^{-3}	milli-	m	thousandth
10^{-6}	micro-	μ	millionth
10^{-9}	nano-	n	billionth
10^{-12}	pico-	p	trillionth

Should know

usually given

Examples:

$$\begin{aligned}
 & 58 \text{ mm} \\
 & = 58 \text{ millimetres} \\
 & = 58 \times 10^{-3} \text{ m}
 \end{aligned}$$

Express 458 μm in km in standard form.

$$\begin{aligned}
 458 \mu\text{m} &= 458 \times 10^{-6} \text{ m} \\
 &= 458 \times 10^{-9} \times 10^3 \text{ m} \quad \text{split up} \\
 &= 458 \times 10^{-9} \text{ km} \\
 &= 4.58 \times 10^{-7} \text{ km}
 \end{aligned}$$

Eg: Light travels 1 metre in 3.33 nanoseconds. Find the speed of light in m/s.

Give your answer in standard form. (1 nanosecond = 10^{-9} second)

$$3.33 \text{ nanoseconds} = 3.33 \times 10^{-9} \text{ seconds}$$

$$\begin{aligned}
 \text{Speed of light} &= \frac{1}{3.33 \times 10^{-9}} \\
 &= 3.00 \times 10^8 \text{ m/s (3 s.f.)}
 \end{aligned}$$

Eg: A high-resolution photograph has a file size of 4.8×10^6 bytes.(a) A solid-state drive (SSD) has a capacity of 512 gigabytes. Given that 1 gigabyte = 1×10^9 bytes, express the capacity of the SSD in bytes in standard form.

$$\begin{aligned}
 \text{Capacity of SSD} &= 512 \text{ gigabytes} \\
 &= 512 \times 10^9 \text{ bytes} \\
 &= \underline{5.12 \times 10^{11} \text{ bytes}}
 \end{aligned}$$

(b) Calculate the maximum number of these photographs that can be stored on the SSD.

Express your answer in standard form, correct to 2 significant figures.

$$\begin{aligned}
 \text{Maximum number of photographs} &= (5.12 \times 10^{11}) \div (4.8 \times 10^6) \\
 &= 1.1 \times 10^5 \text{ (2 s.f.)}
 \end{aligned}$$

5) Simple Interest (Refer to Financial Math Notes for related topics).

$$\text{Interest} = P \times R \times T$$

P : Principal (The starting amount of money borrowed or deposited)

R : interest rate

T : Time in years

$$\text{Total amount} = P \times R \times T + P$$

* per annum means per year.

Eg 1: Hui Xin took a car loan of \$45 000 from a bank which charges a simple interest of 3.5% per annum. The loan is to be repaid over 30 months.

(a) Calculate the total amount of interest Hui Xin has to pay back at the end of 30 months.

$$\begin{aligned} \text{Interest} &= 45\,000 \times \frac{3.5}{100} \times \frac{30}{12} \\ &= \$3937.50 \end{aligned}$$

(b) How much is the monthly instalment?

$$\begin{aligned} \text{Monthly instalment} &= (45\,000 + 3937.50) \div 30 \\ &= \$1631.25 \end{aligned}$$

Eg 2: Sarah received \$562.50 in simple interest after 18 months from an investment at 2.5% per annum. What was her initial investment?

Let Sarah's initial investment be \$P.

$$562.50 = P \times \frac{2.5}{100} \times \frac{18}{12} \quad \leftarrow \text{Form an equation}$$

$$\begin{aligned} P &= 562.50 \div \frac{2.5}{100} \div \frac{18}{12} \\ &= \$15\,000 \end{aligned}$$

Eg 3: Jenny deposited \$3000 in the bank for 5 years. Find the interest rate if her deposit grew to \$3750 after 5 years.

Let the interest rate be $r\%$.

$$3750 - 3000 = 3000 \times \frac{r}{100} \times 5$$

$$750 = 15000 \times \frac{r}{100}$$

$$\frac{r}{100} = \frac{750}{15000}$$

$$r = 0.05 \times 100$$

$$r = 5\%$$

6) Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100}\right)^n$$

$$\text{Interest} = P \left(1 + \frac{r}{100}\right)^n - P$$

Given in math formula

P : Principal (The starting amount of money)

r : interest rate

n : number of times the interest is compounded

→ annually: compounded once a year

→ half-yearly: compounded twice a year

→ quarterly: compounded 4 times a year

→ monthly: compounded 12 times a year

Eg 1: Mrs Lee deposited \$15 000 in an account that pays 2.68% compound interest per annum. Find the total amount in the account after 5 years if the interest is compounded **monthly**.

$$\begin{aligned} \text{Total amount} &= 15\,000 \left(1 + \frac{2.68 \div 12}{100}\right)^{5 \times 12} \quad [\text{inside divide, outside times}] \\ &= \$17\,148.33 \quad (2 \text{ d.p.}) \end{aligned}$$

Eg 2: Li Ya places \$6000 in a bank that offers an interest compounded **quarterly**. At the end of 3 years, she received \$964.53 in interest. Find the interest rate offered by the bank.

$$\text{Interest} = P \left(1 + \frac{r}{100}\right)^n - P$$

$$964.53 = 6000 \left(1 + \frac{r \div 4}{100}\right)^{3 \times 4} - 6000$$

$$964.53 + 6000 = 6000 \left(1 + \frac{r \div 4}{100}\right)^{12}$$

$$\frac{6964.53}{6000} = \left(1 + \frac{r \div 4}{100}\right)^{12}$$

$$\sqrt[12]{\frac{6964.53}{6000}} = 1 + \frac{r \div 4}{100}$$

$$\frac{r \div 4}{100} = \sqrt[12]{\frac{6964.53}{6000}} - 1$$

$$r \div 4 = \left(\sqrt[12]{\frac{6964.53}{6000}} - 1\right) \times 100$$

$$r = \left(\sqrt[12]{\frac{6964.53}{6000}} - 1\right) \times 100 \times 4$$

$$= 5.00\% \quad (3 \text{ s.f.})$$

“shift” 6000 to LHS

÷ 6000 on both sides

root 12 on both sides

“shift” +1 to LHS. Then, swap LHS and RHS

× 100 on both sides

× 4 on both sides

Key into calculator 