Sec 1 Chapter 1 – Primes, HCF and LCM

Must know questions:

1 (a) Express 3969 as a product of its prime factors in index notation.

(b) Hence, find the value of $\sqrt{3969}$.

(c) Find the smallest value of k such that 3969k is a perfect cube.

2 Given that $2646 = 2 \times 3^3 \times 7^2$,
(a) Is 2646 a perfect cube? Explain your answer.

(b) Find the smallest possible value of m and n such that $\frac{2646m}{n}$ is a perfect square, where $n > m$.

3 Written as a product of its prime factors, $600 = 2^3 \times 3 \times 5^2$.
(a) Find the largest integer that can divide both 135 and 600.

(b) Find the smallest positive integer n for which $600n$ is a multiple of 54.

4 The highest common factor and lowest common multiple of two numbers are 42 and 252 respectively. The two numbers are greater than 50. Find the two numbers.

5 Mrs Lim bought 56 pencils, 84 erasers and 112 notebooks for her students. She plans to distribute the items equally to her students.

(a) Find the largest number of students in her class.

(b) How many of each item will each student receive?

6 The school bell rings every 9 minutes, and the cafeteria bell rings every 12 minutes. If they both ring together for the first time at 08 35, at what time will they next ring together?

Sec 1 Chapter 1 – Primes, HCF and LCM

Must know questions:

1 (a) Express 3969 as a product of its prime factors in index notation.

* Take note of what the question wants

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

$3969 = 3^4 \times 7^2$

Ans: $3^4 \times 7^2$

(b) Hence, find the value of $\sqrt{3969}$.

$\sqrt{3969} = \sqrt{3^4 \times 7^2}$

$= 3^2 \times 7$

$= 63$

Ans: 63

(c) Find the smallest value of k such that 3969k is a perfect cube.

↳ powers of prime factors must be multiple of 3

$3969K = 3^4 \times 7^2 \times K$

$K = 3^2 \times 7$ ← To make the powers become multiple of 3

$= 63$

Ans: 63

2 Given that $2646 = 2 \times 3^3 \times 7^2$,

(a) Is 2646 a perfect cube? Explain your answer.

No. Not all the indices / powers of the prime factors are multiples of 3, hence 2646 is not a perfect cube.

Note: singular: index | plural: indices or "powers"

2^3 ← index, ↑ base

(b) Find the smallest possible value of m and n such that $\frac{2646m}{n}$ is a perfect square, where $n > m$.

↳ powers must be multiples of 2/even.

$\frac{2646m}{n} = \frac{2 \times 3^3 \times 7^2 \times m}{n}$ ← multiplying by m will increase powers; ↳ dividing by n will decrease powers

$\frac{m}{n} = \frac{2}{3}$

Ans: $m = 2$, $n = 3$

3 Written as a product of its prime factors, $600 = 2^3 \times 3 \times 5^2$.

(a) Find the largest integer that can divide both 135 and 600.

↳ This means find HCF

3	135
3	45
3	15
5	5
	1

$135 = 3^3 \times 5$

$600 = 2^3 \times 3 \times 5^2$

$HCF = 3 \times 5$

$= 15$

Ans: 15

(b) Find the smallest positive integer n for which $600n$ is a multiple of 54.

↳ 600n is the LCM of 54 and 600

2	54
3	27
3	9
3	3
	1

$54 = 2 \times 3^3$

$600 = 2^3 \times 3 \times 5^2$

$LCM = 2^3 \times 3^3 \times 5^2 = 600 \times n$

$n = 3^2$

$= 9$

Ans: 9

4 The highest common factor and lowest common multiple of two numbers are 42 and 252 respectively. The two numbers are greater than 50. Find the two numbers.

Step 1 prime factorise the HCF & LCM

HCF = 42 = $2 \times 3 \times 7$

LCM = 252 = $2^2 \times 3^2 \times 7$

2	42
3	21
7	7
	1

2	252
2	126
3	63
3	21
7	7
	1

Step 2 Write the HCF prime factors for both numbers.

1st number = $2^2 \times 3 \times 7 = 84$

2nd number = $2 \times 3^2 \times 7 = 126$

Step 3

Ensure LCM's prime factors are "found" in either number. Check that both numbers are greater than 50.

Ans: 84 and 126

5 Mrs Lim bought 56 pencils, 84 erasers and 112 notebooks for her students. She plans to distribute the items equally to her students.

(a) Find the largest number of students in her class. ↳ HCF

Method 1

For HCF only divide by common prime factors for all numbers

	P	E	N
2	56,	84,	112
2	28,	42,	56
7	14,	21,	28
	(2),	(3),	(4)

HCF = $2^2 \times 7$
= 28

Method 2

$56 = 2^3 \times 7$

$84 = 2^2 \times 3 \times 7$

$112 = 2^4 \times 7$

HCF = $2^2 \times 7$
= 28

Ans: 28

(b) How many of each item will each student receive?

Method 1

Ans: 2 pencils
3 Erasers
4 Notebooks

Method 2

pencils = $56 \div 28$
= 2

Erasers = $84 \div 28$
= 3

Notebooks = $112 \div 28$
= 4

6 The school bell rings every 9 minutes, and the cafeteria bell rings every 12 minutes. If they both ring together for the first time at 08 35, at what time will they next ring together? * LCM

Method 1

For LCM continue dividing by prime factors for each number

3	9,	12
3	3,	4
2	1,	4
2	1,	2
	1,	1

LCM = $2^2 \times 3^2$
= 36 min

Method 2

$9 = 3^2$

$12 = 2^2 \times 3$

LCM = $2^2 \times 3^2$
= 36 min

0835 → (36 min) → 09 11

Ans: 09 11