CHAPTER 11 AM Differentiation Techniques

1) Notation

Question	Differentiate... OR Find the derivative of...	$5x^2$	$y = \frac{1}{2x}$	$f(x) = 3x^3 + 8$
How to present	1^{st} derivative (differentiate once)	$\frac{d}{dx}(5x^2) = 10x$	$\frac{dy}{dx} = -\frac{1}{x^2}$	$f'(x) = 9x^2$
	2^{nd} derivative (differentiate twice)	$\frac{d^2}{dx}(5x^2) = 10$	$\frac{d^2y}{dx^2} = \frac{1}{x^3}$	$f''(x) = 18x$

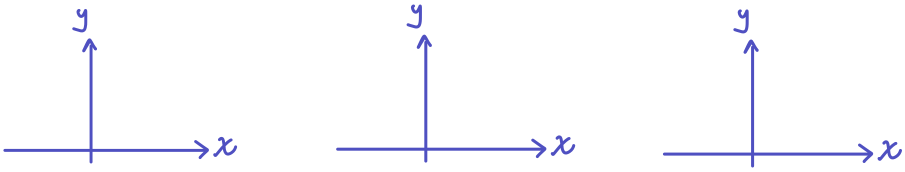
2) What does $\frac{dy}{dx}$ represent?

→ The ______________________ to the graph of $y = f(x)$ at any point

→ The instantaneous ______________________ with respect to x

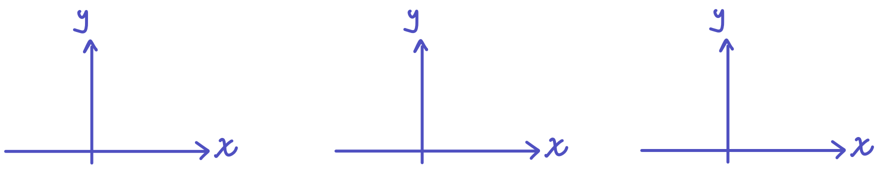
→ if $\frac{dy}{dx} > 0$, then the curve $y = f(x)$ is __________

Eg:



→ if $\frac{dy}{dx} < 0$, then the curve $y = f(x)$ is __________

Eg:



3) What does $\frac{d^2y}{dx^2}$ represent?

→ The ______________________ (how the gradient changes)

4) Tips (before differentiating)

✓ Remove roots (indices recap: $\sqrt[3]{x^2} = x^{\frac{2}{3}}$)

✓ Remove fractions (unless using quotient rule)

✓ If 'x' is in the denominator, move it up!

Eg: $\frac{2}{3\sqrt{x}} = \frac{2}{3x^{\frac{1}{2}}} = \frac{2}{3}x^{-\frac{1}{2}}$

5) Algebraic Functions (Bring down power, power − 1)

('a' is a constant)

EXAMPLES

$$\boxed{\frac{d}{dx}(a) = 0}$$

$\frac{d}{dx}(\pi) =$

$\frac{d}{dx}\left(-\frac{\sqrt{3}}{2}\right) =$

$$\boxed{\frac{d}{dx}(ax^n) = anx^{n-1}}$$

$\frac{d}{dx}\left(\frac{1}{2}x^{-3}\right) =$

Multiple terms ⟶ differentiate each term independently

$$\boxed{\frac{d}{dx}(ax^n + bx^m + \ldots) = anx^{n-1} + bmx^{m-1} + \ldots}$$

$y = \frac{4}{9x^3} + 5x\sqrt{x} - \frac{3}{\sqrt[3]{x^2}}$

$\frac{dy}{dx} =$

6) Chain Rule (Differentiate outside × differentiate inside)

$$\boxed{\frac{d}{dx}[(ax+b)^n] = n(ax+b)^{n-1}(a)}$$

$y = \sqrt[3]{2x^2 + 7}$

$\frac{dy}{dx} =$

7) Product Rule (copy 1st term × diff 2nd term + copy 2nd term × diff 1st term)

$$\boxed{\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$y = (2x+3)(x-3)^4$

$\frac{dy}{dx} =$

8) Quotient Rule $\left(\frac{\text{copy bottom} \times \text{diff. top} - \text{copy top} \times \text{diff. bottom}}{\text{bottom}^2}\right)$

$$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}}$$

$y = \frac{5x + 9}{(1 - 3x^2)}$

$\frac{dy}{dx} =$

9) Increasing and decreasing functions

a) The curve is an increasing / decreasing function

Eg 1: Finding the value(s) / range of unknown

Find the set of values of x for which $f(x) = 2x^3 - 7x^2 + 4x + 3$ is an increasing function.

Eg 2: Showing / Proving questions

The equation of the curve is $y = -x^3 + 3x^2 - 6x + 5$. Show that the curve is always decreasing for all real values of x.

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b) The gradient of curve is an increasing or decreasing function.

Eg: The derivative of a function is given by $f'(x) = \frac{3x-1}{x+2}$ for $x > 0$.

Determine if the gradient of the curve is an increasing or decreasing function.

Example questions

Eg 1:

Find the coordinates of the point(s) on the curve $y = 2x^3 + 2x^2 - 5$ where the tangent is perpendicular to the line $2y - 2 - 4x = 0$.

Eg 2:

The equation of a curve is $y = \frac{5}{(1-3x)^2} - 12$

(a) Find the first and second derivatives.

(b) Find the gradient of the curve at the y-axis.