# CHAPTER 3 Algebra: Expansion & Factorisation

## 1) Basics: Recap: Sec 1 Chap 4

- $2x + 3x = 5x$
- $-3x + 2x = -x$
- $5x + y - 3x = 2x + y$
- $-(a+b+c) = -a-b-c$
- $-(a-b-c) = -a+b+c$
- $x - y = -(y - x)$

- $0 \times y = 0$
- $1 \times y = y$
- $2(y) = 2y = y + y$
- $x^2y = yx^2 \neq xy^2$

- $y \times y = y^2$
- $(-y)^2 = y^2$
- $-y^2 = -y^2$
- $-(-y)^2 = -y^2$
- $(2y)^2 = 4y^2$
- $2y^2 = 2y^2$
- $y \times y \times y = y \times y^2 = y^3$
- $(-y)^3 = (-y)(-y)(-y) = -y^3$

## 2) Expansion (rainbow method)

$(a+b)(x+y) = ax + ay + bx + by$

Eg: Expand and simplify the following

(a) $(2a+b)(4a-7)$

$= 8a^2 - 14a + 4ab - 7b$

(b) $(4x-5y)(-10-6y)$

$= -40x - 24xy + 50y + 30y^2$

(c) $-(7+8a)(-3b+c)$

$= -(-21b + 7c - 24ab + 8ac)$

$= 21b - 7c + 24ab - 8ac$

(d) $(-3x+8-10y)(2x+y)$

$= -6x^2 - 3xy + 16x + 8y - 20xy - 10y^2$

$= -6x^2 - 23xy + 16x + 8y - 10y^2$

(e) $x^2 + 2xy - (x-3y)(2x+1)$

$= x^2 + 2xy - (2x^2 + x - 6xy - 3y)$

$= x^2 + 2xy - 2x^2 - x + 6xy + 3y$

$= -x^2 + 8xy - x + 3y$

(f) $5(2x-7) - (3x-2)(x+1)$

$= 10x - 35 - (3x^2 + 3x - 2x - 2)$

$= 10x - 35 - 3x^2 - 3x + 2x + 2$

$= -3x^2 + 9x - 33$

(g) $(h+3k)(4h-7) - (3k-h)(2+5h)$

$= 4h^2 - 7h + 12hk - 21k - (6k + 15hk - 2h - 5h^2)$

$= 4h^2 - 7h + 12hk - 21k - 6k - 15hk + 2h + 5h^2$

$= 9h^2 - 5h - 3hk - 27k$

# 2) Factorisation ("take out" HCF) (Recap: Sec 1 Chap 4)

## a) Multiplication frame

- Used to factorise quadratic expressions $(ax^2 + bx + c)$

> ! Quadratic expressions / equations: The highest power of the variable is 2.

| × | A | B |
|---|---|---|
| C | Box 1 | Box 2 |
| D | Box 3 | Box 4 |

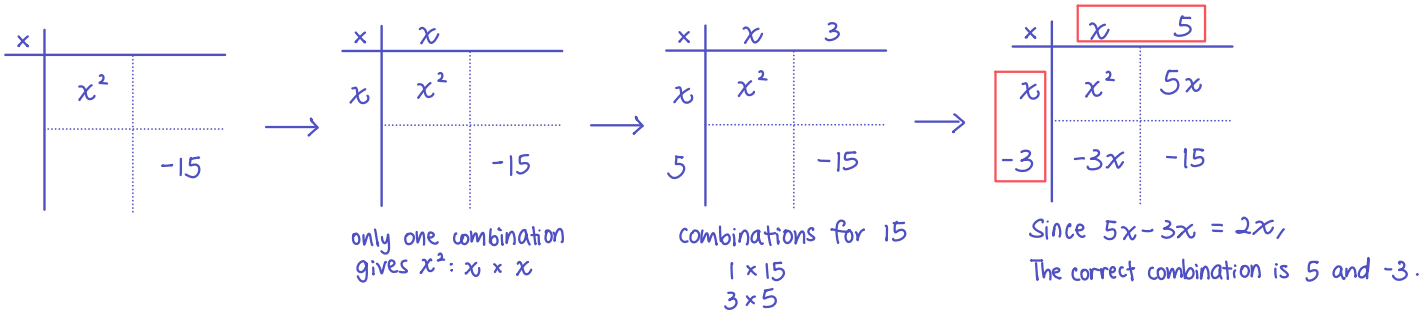
① In the frame, fill up Box 1 and Box 4 first. Write with the +/- sign.
- Box 1 → $ax^2$ (Eg: $5x^2$)
- Box 4 → constant (Eg: $-20$)

② Find the combinations where
A × C = Box 1
B × D = Box 4

③ Fill up Box 2 and Box 3, where
B × C = Box 2
A × D = Box 3

④ Check if the combination is correct, such that
Box 2 + Box 3 = $bx$

**Eg: Factorise $x^2 + 2x - 15$**




$\therefore x^2 + 2x - 15 = (x+5)(x-3)$

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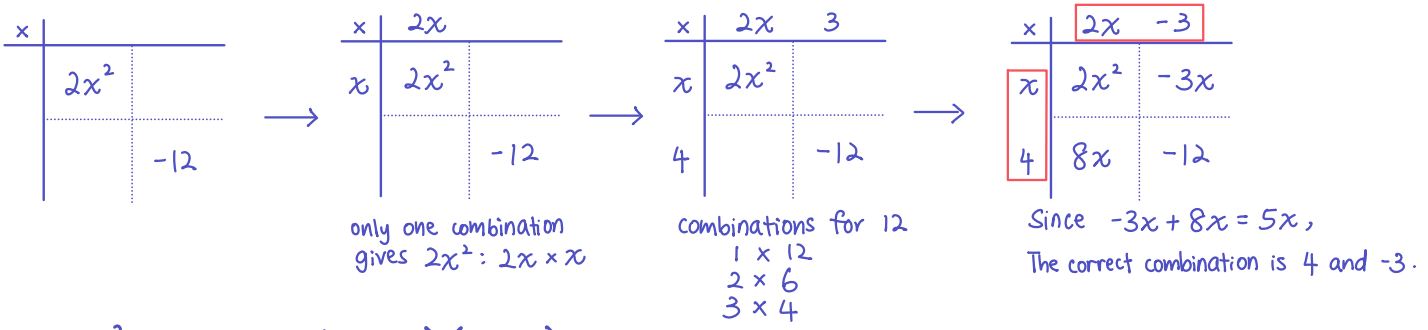
**Eg: Factorise $2x^2 + 5x - 12$**




$\therefore 2x^2 + 5x - 12 = (2x-3)(x+4)$

Calculator Hack!

① menu → 5 → 2 → 2

② Key in the question: Eg: $2x^2 + 5x - 12$
2 → = → 5 → = → -12 → = → =

③ Calculator shows $\boxed{x = \frac{3}{2}}$. Press '=' again. $\boxed{x = -4}$

④ (a) Get rid of fraction by multiplying both sides by the denominator.
(b) "Shift" the number to the left.

$x = \frac{3}{2}$
$2x = 3$
$2x - 3 = 0$

$x = -4$
$x + 4 = 0$

These are the two factors.

Eg : Factorise $8 - 3x^2 - 5x$ ↻ Rearrange

$= -3x^2 - 5x + 8$

$= (3x + 8)(1 - x)$

| × | $3x$ | $8$ |
|---|---|---|
| $-x$ | $-3x^2$ | $-8x$ |
| $1$ | $3x$ | $8$ |

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Eg : Factorise $-2y^2 - 2y + 12$ Always factorise the HCF of all terms first

$= -2(y^2 + y - 6)$

$= -2(y + 3)(y - 2)$

| × | $y$ | $3$ |
|---|---|---|
| $y$ | $y^2$ | $3y$ |
| $-2$ | $-2y$ | $-6$ |

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Eg : Factorise $m^2 + 7mn - 8n^2$

| × | $m$ | $8n$ |
|---|---|---|
| $m$ | $m^2$ | $8mn$ |
| $-n$ | $-mn$ | $-8n^2$ |

Fill the two terms with the square ($m^2$ and $-8n^2$) into Box 1 and 4.

$\therefore m^2 + 7mn - 8n^2 = (m + 8n)(m - n)$

Eg : Factorise $5g^2 + 7gh - 6h^2$

| × | $5g$ | $-3h$ |
|---|---|---|
| $g$ | $5g^2$ | $-3gh$ |
| $2h$ | $10gh$ | $-6h^2$ |

$\therefore 5g^2 + 7gh - 6h^2 = (5g - 3h)(g - 2h)$

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## Common factorisation questions

Eg 1 : [Source : Fairfield Methodist Secondary school / 2024 / WA2 / Q6]

(a) Factorise $2p^2 + 13p - 7$

$2p^2 + 13p - 7 = (2p - 1)(p + 7)$

| × | $2p$ | $-1$ |
|---|---|---|
| $p$ | $2p^2$ | $-p$ |
| $7$ | $14p$ | $-7$ |

(b) Hence, factorise $2(x - y)^2 + 13(x - y) - 7$

Using (a), substitute $p = x - y$

$2(x - y)^2 + 13(x - y) - 7 = [2(x - y) - 1](x - y + 7)$

$= (2x - 2y - 1)(x - y + 7)$

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Eg 2 : (a) Factorise $n^2 + 7n + 12$.

$n^2 + 7n + 12 = (n + 4)(n + 3)$

| × | $n$ | $4$ |
|---|---|---|
| $n$ | $n^2$ | $4n$ |
| $3$ | $3n$ | $12$ |

(b) Hence, find two factors of 182 by using substitution for n.

Substitute $n = 10$

$182 = 10^2 + 7(10) + 12$

$= (10 + 4)(10 + 3)$

$= 14 \times 13$

$\therefore$ The two factors are 13 and 14.

## b) Grouping

- used to factorise 4 terms

① Group terms into 2 pairs. Rearrange the terms if needed.
② For each pair, take out the common factor.
③ If the brackets are not the same, use tip: $(y-x) = -(y-x)$ → ! Keep, Change, Flip
④ Take out the common bracket.
⑤ Ensure all brackets are fully factorised.

Eg: Factorise the following.

(a) $\overbrace{3a(y+3)}^{\text{1st term}} - \overbrace{4(y+3)}^{\text{2nd term}}$

$= (y+3)(3a-4)$ ④

(b) $3(y-2) + 5a(2-y)$ $(2-y) = -(y-2)$

Keep, change, flip

$= 3(y-2) - 5a(y-2)$ ③

$= (y-2)(3-5a)$ ④

(c) $3pq - 6sq - 2p + 4s$

$= 3q(p-2s) - 2(p-2s)$ ②

$= (p-2s)(3q-2)$ ④

(d) $9m + 12n - 3mp - 4np$

$= 9m - 3mp + 12n - 4np$ ①

$= 3m(3-p) + 4n(3-p)$ ②

$= (3-p)(3m+4n)$ ④

(e) $7r + 6s - 3sr - 14$

$= 7r - 14 + 6s - 3sr$ ①

$= 7(r-2) + 3s(2-r)$ ②

$= 7(r-2) - 3s(r-2)$ ③

$= (r-2)(7-3s)$ ④

(f) $24ab + 18ac - 8bd - 6dc$

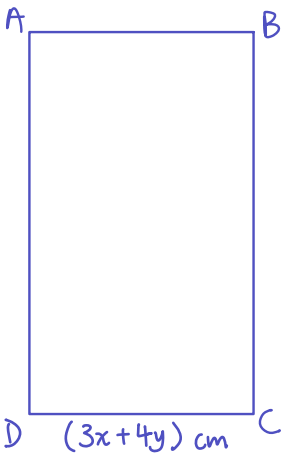
$= 6a(4b+3c) - 2d(4b+3c)$ ②

$= (4b+3c)(6a-2d)$ ④ not fully factorised

$= 2(4b+3c)(3a-d)$ ⑤

## 3) Application question

Eg: The area of rectangle ABCD is $(6x^2 + 23xy + 20y^2)\ cm^2$. Find the perimeter of ABCD.




$6x^2 + 23xy + 20y^2 = (3x+4y)(2x+5y)$

| x | 3x | 4y |
|---|---|---|
| 2x | $6x^2$ | 8xy |
| 5y | 15xy | $20y^2$ |

Length of BC = $(2x+5y)$ cm

Perimeter $= 2(2x+5y) + 2(3x+4y)$
$= 4x + 10y + 6x + 8y$
$= (10x + 18y)$ cm