Chapter 1: Primes, HCF & LCM

1) Prime number : a whole number that has exactly 2 different factors, 1 and itself.

Eg: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 ... (no negative numbers)

2) Composite number : a whole number that has more than 2 different factors.

Eg: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 ... (no negative numbers)

> **Note:**
> 0 and 1 are neither prime nor composite

3) Prime factorisation means writing a composite number as a product of its prime factors.

Eg: Find the prime factorisation of 24.
Eg: Express 24 as a product of its prime factors in index notation.
} same question, asked differently.

Start with the smallest prime number

2	24	
2	12	← $24 \div 2$
2	6	
3	3	
	1	← Stop when we reach 1

This means "therefore" → $\therefore 24 = 2^3 \times 3$

4) Perfect square :

- A whole number whose square root is a whole number. (use calculator to check)
- A whole number that can be written as a whole number × the same whole number.
- A whole number in which the indices of its prime factors are all multiples of 2 / even.

Eg: $0, 1, 4, 9, 16, a^2, b^4, 2^4 \times 7^6$ (no negative numbers)

5) Perfect cube :

- A whole number whose cube root is a whole number. (use calculator to check)
- A whole number in which the indices of its prime factors are all multiples of 3.

Eg: $-125, -1, 0, 1, 8, 27, 4^3, x^9, 5^{15} \times 11^6$

Eg: 7^3
- 3 ← index (Singular: index | Plural: indices) ("power")
- 7 ← base

6) Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

Eg: Find the HCF and LCM of 36, 54, 90

Method 1

common prime factors

$$\begin{array}{c|ccccc} 2 & 36, & 54, & 90 \\ \hline 3 & 18, & 27, & 45 \\ \hline 3 & 6, & 9, & 15 \\ \hline & 2, & 3, & 5 \end{array}$$

← Divide each number by 2

← Stop when there is no common prime factor to divide

$\therefore$ HCF $= 2 \times 3^2$
$= 18$

and

LCM $= 2 \times 3^2 \times 2 \times 3 \times 5$
$= 2^2 \times 3^3 \times 5$
$= 540$

Method 2

$36 = 2^2 \times 3^2$
$54 = 2 \times 3^3$
$90 = 2 \times 3^2 \times 5$

Step ①: Express each number in its prime factors

$\therefore$ HCF $= 2 \times 3^2$
$= 18$

Step ② (HCF): ✓ Choose common prime factors (2, 3)
✓ Choose the lowest power for each prime factor

LCM $= 2^2 \times 3^3 \times 5$
$= 540$

Step ② (LCM): ✓ Choose all prime factors (2, 3, 5
✓ Choose the highest power for each prime factor.

7) Tips when solving real-world context questions:

When unsure whether to find LCM or HCF, look at what you need to find first.

- If you need an answer smaller than the given numbers ⟶ Find HCF
  (eg: cutting tiles, dividing groups, splitting items)

- If you need an answer larger than the given numbers ⟶ Find LCM
  (eg: finding a future date, next meeting time)

Sec 1 Chapter 1 – Primes, HCF and LCM

Must know questions:

1 (a) Express 3969 as a product of its prime factors in index notation.

*Take note of what the question wants

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

$3969 = 3^4 \times 7^2$

Ans: $3^4 \times 7^2$

(b) Hence, find the value of $\sqrt{3969}$.

$\sqrt{3969} = \sqrt{3^4 \times 7^2}$

$= 3^2 \times 7$

$= 63$

Ans: 63

(c) Find the smallest value of k such that 3969k is a perfect cube.

↳ powers of prime factors must be multiple of 3

$3969K = 3^4 \times 7^2 \times K$

$K = 3^2 \times 7$ ← To make the powers become multiple of 3

$= 63$

Ans: 63

2 Given that $2646 = 2 \times 3^3 \times 7^2$,

(a) Is 2646 a perfect cube? Explain your answer.

No. Not all the indices / powers of the prime factors are multiples of 3, hence 2646 is not a perfect cube.

Note: 2^3 ← 3 index; ↑ base

singular	plural
index	indices

or "powers"

(b) Find the smallest possible value of m and n such that $\frac{2646m}{n}$ is a perfect square, where $n > m$.

↳ powers must be multiples of 2/even.

$\frac{2646m}{n} = \frac{2 \times 3^3 \times 7^2 \times m}{n}$ ← multiplying by m will increase powers; ↳ dividing by n will decrease powers

$\frac{m}{n} = \frac{2}{3}$

Ans: $m = 2$, $n = 3$

3 Written as a product of its prime factors, $600 = 2^3 \times 3 \times 5^2$.

(a) Find the largest integer that can divide both 135 and 600.

↳ This means find HCF

3	135
3	45
3	15
5	5
	1

$135 = 3^3 \times 5$

$600 = 2^3 \times 3 \times 5^2$

$HCF = 3 \times 5$
$= 15$

Ans: 15

(b) Find the smallest positive integer n for which $600n$ is a multiple of 54.

↳ $600n$ is the LCM of 54 and 600

2	54
3	27
3	9
3	3
	1

$54 = 2 \times 3^3$

$600 = 2^3 \times 3 \times 5^2$

$LCM = 2^3 \times 3^3 \times 5^2 = 600 \times n$

$n = 3^2$
$= 9$

Ans: 9

4 The highest common factor and lowest common multiple of two numbers are 42 and 252 respectively. The two numbers are greater than 50. Find the two numbers.

Step 1
prime factorise the HCF & LCM

$HCF = 42 = 2 \times 3 \times 7$

$LCM = 252 = 2^2 \times 3^2 \times 7$

2	42
3	21
7	7
	1

2	252
2	126
3	63
3	21
7	7
	1

Step 2
Write the HCF $2 \times 3 \times 7$ for both numbers.

1st number $= 2^2 \times 3 \times 7 = 84$

2nd number $= 2 \times 3^2 \times 7 = 126$

Step 3
Ensure LCM's prime factors are "found" in either number. Check that both numbers are greater than 50.

Ans: 84 and 126

5 Mrs Lim bought 56 pencils, 84 erasers and 112 notebooks for her students. She plans to distribute the items equally to her students.

(a) Find the largest number of students in her class.

↳ HCF

Method 1

For HCF only divide by common prime factors for all numbers

	P	E	N
2	56,	84,	112
2	28,	42,	56
7	14,	21,	28
	2,	3,	4

$HCF = 2^2 \times 7$
$= 28$

Method 2

$56 = 2^3 \times 7$
$84 = 2^2 \times 3 \times 7$
$112 = 2^4 \times 7$
$HCF = 2^2 \times 7$
$= 28$

Ans : 28

(b) How many of each item will each student receive?

Method 1

Ans : 2 pencils
3 Erasers
4 Notebooks

Method 2

pencils $= 56 \div 28$
$= 2$
Erasers $= 84 \div 28$
$= 3$
Notebooks $= 112 \div 28$
$= 4$

6 The school bell rings every 9 minutes, and the cafeteria bell rings every 12 minutes. If they both ring together for the first time at 08 35, at what time will they next ring together? * LCM

Method 1

For LCM continue dividing by prime factors for each number

3	9,	12
3	3,	4
2	1,	4
2	1,	2
	1,	1

$LCM = 2^2 \times 3^2$
$= 36$ min

Method 2

$9 = 3^2$
$12 = 2^2 \times 3$
$LCM = 2^2 \times 3^2$
$= 36$ min

$0835 \xrightarrow{36\text{ min}} 09\ 11$

Ans : 09 11

7 A rectangular function room has dimensions 1260 cm by 1080 cm.
A worker needs to cover the entire floor with the smallest number of identical square carpet tiles.
(a) Find the length of the side of each square carpet tile.

don't get tricked by this word! It does not mean LCM.

$\boxed{\text{TIP}}$: When unsure whether to find LCM or HCF, ask yourself : "Will my answer be smaller or bigger than the given numbers in the question?"
- If smaller ⟶ HCF
- If bigger ⟶ LCM

2	1260
2	630
3	315
3	105
5	35
7	7
	1

2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

$1260 = 2^2 \times 3^2 \times 5 \times 7$

$1080 = 2^3 \times 3^3 \times 5$

$HCF = 2^2 \times 3^2 \times 5$

$= 180$

$\therefore$ The length of the side of each square carpet tile is 180 cm.

(b) How many of square carpet tiles will the worker use in total?

$1260 \div 180 = 7$

$1080 \div 180 = 6$

$7 \times 6 = 42$

The worker will use 42 square carpet tiles.