CHAPTER 3 Approximation and Estimation

Legend : d.p. : decimal place(s)
s.f. : significant figure(s)

1) Rounding off whole numbers and decimals (Recap of primary school)

Step 1 : Find the digit at the place you are rounding to.
Step 2 : Look at the next digit to its right.
Step 3 : • If it is 5 or more → add 1 to the previous digit (round up)
• If it is less than 5 → keep the previous digit the same (round down)
Step 4 : Remove the remaining digits (for decimals) or add zero (for whole numbers)

Eg: Round 5.3761 to 2 decimal places.

5.3761 = 5.38 (2d.p.)

→ Find the 2nd decimal place, which is 7
→ The digit right after it is 6 → round up
→ Remove remaining digits

Eg: Round 673213 to the nearest 100.

673 213 = 673 200 (nearest 100)

→ Find the digit in the hundreds place → 2
→ The digit right after it is 1 → round down
→ Add zeros at the back

2) 5 rules of significant figures :

Rule 1	All non-zero digits are significant
eg:	323 (3s.f.) , 78.23 (4 s.f.)
Rule 2	All zeros between non-zero digits are significant.
eg:	303 (3s.f.) , 700.43 (5s.f.)
Rule 3	In a decimal, all zeros before a non-zero digit are not significant
eg:	0.0037 (2 s.f.) , 0.203 (3s.f.)
Rule 4	In a decimal, all zeros after a non-zero digit are significant
eg:	0.00730 (3s.f.) , 0.1230400 (7s.f.)
Rule 5	The zeros at the end of a whole number may or may not be significant, depending on how the number is rounded off
eg:	Consider 8397. If we round to 2 s.f. (or nearest hundred), we get 8400 (2s.f.) If we round to 3 s.f. (or nearest ten), we get 8400 (3s.f.). Hence, the number 8400 could have been either 2 s.f. or 3s.f., depending on how it is rounded off.

3) Steps to round off numbers to given significant numbers

Step 1 : Find the digit at the s.f. you are rounding to, starting from the first non-zero digit.

Step 2 : Look at the next digit to its right.

Step 3 : • If it is 5 or more ⟶ add 1 to the previous digit (round up)

• If it is less than 5 ⟶ Keep the previous digit the same (round down)

Step 4 : Replace the remaining digits with zeros (if needed).

Eg 1: Correct 5.29837 to 3 significant figures.

5.29837 = 5.30 (3 s.f.)

→ Significant digits : 5, 2, 9
→ Next digit is 8 ⟶ round up

Eg 2 : Correct 0.00234017 to 2 significant figures.

0.00234017 = 0.0023 (2 s.f.)

→ Significant digits : 2, 3
→ Next digit is 4 ⟶ round down

Eg 3 : Correct 79 243 to 3 significant figures.

79 243 = 79 200 (3 s.f.)

↑ Add zeros behind

TIP

After rounding, make sure the value is still close to the original value.
The number should stay roughly the same.

Eg : Round 375 to 2 s.f.

✗ 38 (too small ; value changed too much)

✓ 380 (correct ; Add zero)

4) Estimation

Step 1: Round off every number (use " ≈ ")

• General rule : round to the nearest integer or 1 s.f. (unless otherwise stated).

• For square roots $\sqrt{}$ or cube roots $\sqrt[3]{}$, round the number inside to the nearest perfect square or perfect cube eg : $\sqrt{24.7} \approx \sqrt{25}$

→ perfect squares : 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 141 ...

→ perfect cubes : 1, 8, 27, 64, 125, 216 ...

} use your calculator to check!

Step 2 : Calculate (use " = ")

• This step should be simple enough to do without a calculator.

Step 3 : Round final answer according to the question.

Eg 1: Estimate 30.2×0.48.

$$30.2 \times 0.48 \approx 30 \times 0.5$$
$$= 15$$

Here, we round 0.48 to 0.5 as 0.5 is the same as $\frac{1}{2}$, which is easy to mental calculate.

Eg 2: Estimate the value of $\frac{9.8^2 - \sqrt{147}}{1.9 \times \sqrt[3]{67.04}}$, giving your answer correct to 1 significant figure.

$$\frac{9.8^2 - \sqrt{147}}{1.9 \times \sqrt[3]{67.04}} \approx \frac{10^2 - \sqrt{144}}{2 \times \sqrt[3]{64}}$$
$$= \frac{100 - 12}{2 \times 4}$$
$$= \frac{88}{8}$$
$$= 11$$
$$= 10 \text{ (1 s.f.)}$$

* Take note of presentation!
Use $\approx$ for first step when estimating and $=$ for the other steps.

Eg 3: Calculate the value of $\frac{9.8^2 - \sqrt{147}}{1.9 \times \sqrt[3]{67.04}}$, giving your answer correct to 3 significant figures.

$$\frac{9.8^2 - \sqrt{147}}{1.9 \times \sqrt[3]{67.04}} = 10.9 \text{ (3 s.f.)}$$

"Calculate" means you can just key it into your calculator.

Eg 4: By rounding off each number correct to 1 significant figure, estimate the value of $80.4 - 3.31 \times 28.731$.

$$80.4 - 3.31 \times 28.731$$
$$\approx 80 - 3 \times 30$$
$$= 80 - 90$$
$$= -10$$

5) General Rule of Thumb in Mathematics

- Leave your intermediate and final answers in exact form (fractions, surds, π, etc...) when possible.
- If the question does not specify the required accuracy, follow these:

	Final Answer (round to...)	Intermediate steps (truncate to...) → Truncate: copy exactly and cut off
Money	2 d.p.	At least 4 d.p.
Angles	1 d.p. (degrees)	At least 3 d.p.
	3 s.f. (radians)	At least 5 s.f. * Only Sec 3 onwards
Others	3 s.f.	At least 5 s.f.

+2

6) Common Specific Rounding Examples:

Round to nearest...	Given number	Rounded final answer
dollar	\$243.853	\$243
\$10	\$243.853	\$240
\$100	\$243.853	\$200
cent	\$243.853	\$243.85
10 cents	\$243.853	\$243.90
second	54.319 s	54 s
10 seconds	54.319 s	50 s
hour	3.75 h	4 h
whole number	6724.2089	6724
10 000 cm	268 492	270 000
m^3	34 298.51 m^3	34 299 m^3
0.1 m	14.28 m	14.3 m
$\frac{1}{100}$ kg	0.456 kg	0.46 kg