CHAPTER 1 — Simultaneous Linear Equations and Graphs

1) Solving simultaneous equations

↳ Find the value(s) of the unknown. Eg: $x =$ __ , $y =$ __

a) Graphical method

- Plot and draw the 2 linear equations
- The intersection point of the 2 lines will be the solution.

eg. Solve the simultaneous equations.

$$2y - 4 = x$$
$$y = -x + 3$$

$2y - 4 = x$

$2y = x + 4$

① $y = \frac{1}{2}x + 2$

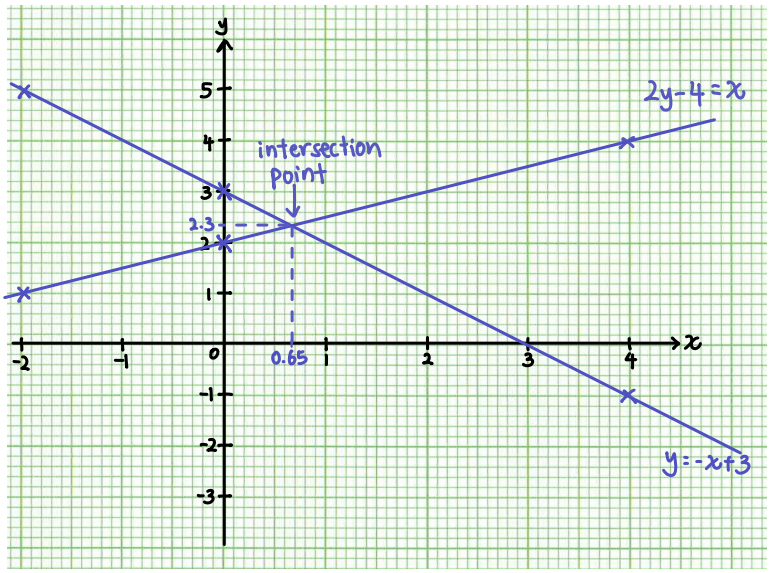
② ③

x	-2	0	4
y	1	2	4

$y = -x + 3$

x	-2	0	4
y	5	3	-1

④



⑤ ∴ The solution is $x = 0.65$ and $y = 2.3$.

Steps

① For each equation, rewrite it into this form: $y = mx + c$. (Make y the subject)

② For each equation, create a table of x and y values. Choose 3 different x-values: a negative x, $x = 0$ and a positive x.

③ Substitute the x-values into the equation to find the corresponding y-values.

④ Plot the 2 lines on your graph paper.

⑤ Find the point where the 2 lines meet. The x- and y- coordinates of this point will be the solution.

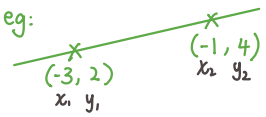
Recap Sec 1 Chap 6 — Equation of straight lines:

$y = mx + c$

- m: Gradient
- c: y-intercept
  - Where the line crosses y-axis.
  - When $x = 0$

• $m = \frac{y_1 - y_2}{x_1 - x_2}$

eg:

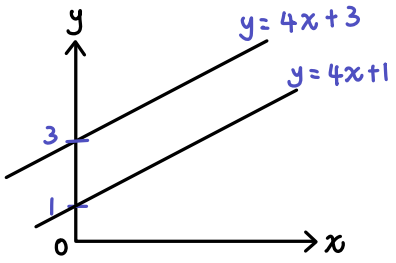
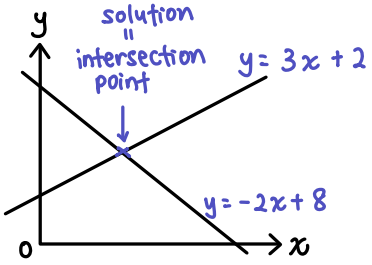
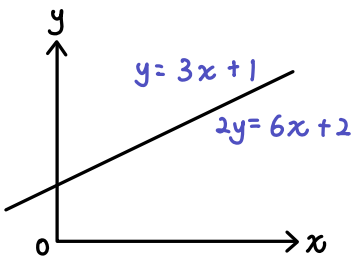


$m = \frac{2 - 4}{-3 - (-1)}$

$= 1$

Graph	Gradient	Example equation
———	$m = 0$	$y = c$ eg: $y = 5$
|	undefined	$x = a$ eg: $x = -3$
/	$m > 0$	$y = 8x + 5$ $y = 11x - 3$
\	$m < 0$	$y = -10x + 9$ $y = 15x - 7$

3 Possible graphs of Simultaneous Equations

① No solution	② 1 solution	③ Infinite number of solutions
		
The two lines are parallel and do not intersect.	The two lines intersect at one point.	The two lines coincide / overlap each other completely.
The two lines have the same gradient, m = 4, and different y-intercepts	The two lines have different gradient.	The two lines have the same gradient and same y-intercept, in which one equation is twice the other.

b) Substitution method

eg. Solve the following pair of simultaneous equations.

$3x + 5y = 11$ ——— (1)

$2x - 4y = 6$ ——— (2)

From (2): $2x = 6 + 4y$

$x = 3 + 2y$ ——— (3)

Sub (3) into (1):

$3(3+2y) + 5y = 11$

$9 + 6y + 5y = 11$

$11y = 2$

$y = \frac{2}{11}$

Sub $y = \frac{2}{11}$ into (3),

$x = 3 + 2\left(\frac{2}{11}\right)$

$= 3\frac{4}{11}$

$\therefore x = 3\frac{4}{11}$, $y = \frac{2}{11}$

Steps

① Look for a variable with the lowest coefficient. In this example, it is '2x'

② Make the chosen variable the subject. In this example, express x in terms of y. (i.e. $x =$ ☐)

③ Substitute into the unused equation (1).

④ Solve for y.

⑤ Substitute y value into (3) to find x

⑥ Check your answers by substituting x and y values back into the original equations

OR Calculator Hack! menu → 5 → 1 → 2

c) Elimination method

eg 1. Solve the following pair of simultaneous equations.

$9y + 2x = 5$ ——— (1)

$3x = 13 + 7y$ ——— (2)

(1) × 3 : $27y + 6x = 15$ ——— (3)

(2) × 2 : $6x = 26 + 14y$ ——— (4)

(3) − (4) : $(27y + 6x) - 6x = 15 - (26 + 14y)$

$27y = -11 - 14y$

$41y = -11$

$y = -\frac{11}{41}$

Sub $y = -\frac{11}{41}$ into (2),

$3x = 13 + 7(-\frac{11}{41})$

$3x = \frac{456}{41}$

$x = 3\frac{29}{41}$

$\therefore x = 3\frac{29}{41}$, $y = -\frac{11}{41}$

Steps

① Compare equations (1) and (3). Pick a variable and make its coefficient the same in both equations by finding the Lowest Common Multiple (LCM). In e.g. 1, pick $2x$ and $3x$ → LCM is $6x$.

② Ensure that the variable you want to eliminate is on the same side in both equations. In e.g. 2, to eliminate y, shift $63y$ to the left-hand side.

③ Eliminate $6x$ by subtracting (4) from (3). OR (3) from (4) in this example.
*Note: Sometimes we add to eliminate variable.

④ Solve for y.

⑤ Substitute y value into either (1) or (2) to find x

⑥ Check your answers by substituting x and y values back into the original equations

OR Calculator Hack!

menu → 5 → 1 → 2 → Key equations → "="

Before you key in your equations, ensure that they are in this form : $ax + by = c$

For example, for this question:

$9y + 2x = 5$

$2x + 9y = 5$ ——— (1)

$3x = 13 + 7y$

$3x - 7y = 13$ ——— (2)

Key in (1) and (2). You will get $x = 3\frac{29}{41}$, $y = -\frac{11}{41}$

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eg 2. Solve the above question by eliminating 'y'.

(1) × 7 : $63y + 14x = 35$ ——— (3)

(2) × 9 : $27x = 117 + 63y$ ↘ Step ②

$-63y + 27x = 117$ ——— (4)

(3) + (4) : $(63y + 14x) + (-63y + 27x) = 35 + 117$

$41x = 152$

$x = 3\frac{29}{41}$

Sub $x = 3\frac{29}{41}$ into (1),

$9y + 2(3\frac{29}{41}) = 5$

$9y = -\frac{99}{41}$

$y = -\frac{11}{41}$

$\therefore x = 3\frac{29}{41}$, $y = -\frac{11}{41}$