Chapter 1: Simultaneous Linear Equations and Graphs

1) Solving simultaneous equations

↳ Find the value(s) of the unknown. Eg: $x =$ __ , $y =$ __

a) Graphical method

- Plot and draw the 2 linear equations
- The intersection point of the 2 lines will be the solution.

eg. Solve the simultaneous equations.

$$2y - 4 = x$$
$$y = -x + 3$$

Steps

1. For each equation, rewrite it into this form: $y = mx + c$. (Make y the subject)
2. For each equation, create a table of x and y values. Choose 3 different x-values: a negative x, $x = 0$ and a positive x.
3. Substitute the x-values into the equation to find the corresponding y-values.
4. Plot the 2 lines on your graph paper.
5. Find the point where the 2 lines meet. The x- and y- coordinates of this point will be the solution.

$2y - 4 = x$

$2y = x + 4$

① $y = \frac{1}{2}x + 2$

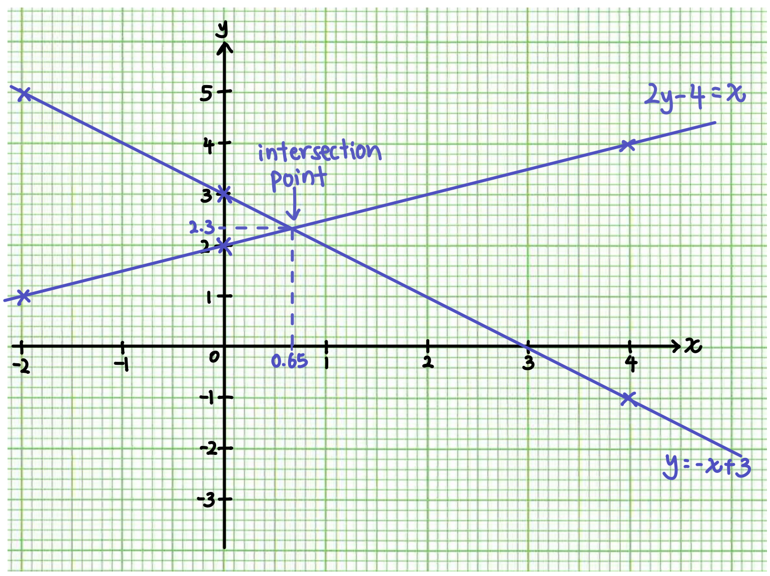
$y = -x + 3$

② ③

x	-2	0	4
y	1	2	4

x	-2	0	4
y	5	3	-1

④



⑤ $\therefore$ The solution is $x = 0.65$ and $y = 2.3$.

Recap Sec 1 Chap 6

Equation of straight lines:

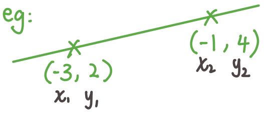
$$y = mx + c$$

Gradient: m

y-intercept: c

- $m = \frac{y_1 - y_2}{x_1 - x_2}$

eg:



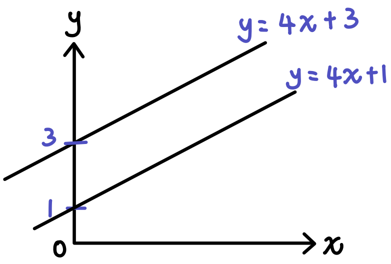
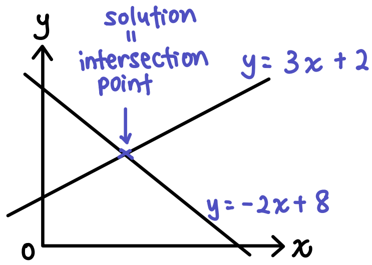
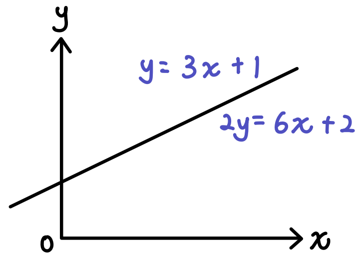
$m = \frac{2 - 4}{-3 - (-1)}$

$= 1$

- Where the line crosses y-axis.
- When $x = 0$

Graph	Gradient	Example equation
	$m = 0$	$y = c$ eg: $y = 5$
	undefined	$x = a$ eg: $x = -3$
	$m > 0$	$y = 8x + 5$ $y = 11x - 3$
	$m < 0$	$y = -10x + 9$ $y = 15x - 7$

3 Possible graphs of Simultaneous Equations

① No solution	② 1 solution	③ Infinite number of solutions
		
The two lines are parallel and do not intersect.	The two lines intersect at one point.	The two lines coincide / overlap each other completely.
The two lines have the same gradient, $m=4$, and different y-intercepts	The two lines have different gradient.	The two lines have the same gradient and same y-intercept, in which one equation is twice the other.

b) Substitution method

eg. Solve the following pair of simultaneous equations.

$$3x + 5y = 11 \quad \text{———} \quad (1)$$
$$2x - 4y = 6 \quad \text{———} \quad (2)$$

From (2): $2x = 6 + 4y$

$$x = 3 + 2y \quad \text{———} \quad (3)$$

Sub (3) into (1):

$$3(3+2y) + 5y = 11$$
$$9 + 6y + 5y = 11$$
$$11y = 2$$
$$y = \frac{2}{11}$$

Sub $y = \frac{2}{11}$ into (3),

$$x = 3 + 2\left(\frac{2}{11}\right)$$
$$= 3\frac{4}{11}$$

$$\therefore x = 3\frac{4}{11}, \quad y = \frac{2}{11}$$

Steps

① Look for a variable with the lowest coefficient. In this example, it is '$2x$'.

② Make the chosen variable the subject. In this example, express x in terms of y. (i.e. $x = \boxed{}$)

③ Substitute into the unused equation (1).

④ Solve for y.

⑤ Substitute y value into (3) to find x

⑥ Check your answers by substituting x and y values back into the original equations

OR Calculator Hack! menu → 5 → 1 → 2

c) Elimination method

eg 1. Solve the following pair of simultaneous equations.

$$9y + 2x = 5 \quad \text{———} \quad (1)$$
$$3x = 13 + 7y \quad \text{———} \quad (2)$$

$(1) \times 3$: $27y + 6x = 15$ ——— (3)

$(2) \times 2$: $6x = 26 + 14y$ ——— (4)

$(3) - (4)$: $(27y + 6x) - 6x = 15 - (26 + 14y)$

$$27y = -11 - 14y$$
$$41y = -11$$
$$y = -\frac{11}{41}$$

Sub $y = -\frac{11}{41}$ into (2),

$$3x = 13 + 7\left(-\frac{11}{41}\right)$$
$$3x = \frac{456}{41}$$
$$x = 3\frac{29}{41}$$

$$\therefore x = 3\frac{29}{41}, \quad y = -\frac{11}{41}$$

eg 2. Solve the above question by eliminating 'y'.

$(1) \times 7$: $63y + 14x = 35$ ——— (3)

$(2) \times 9$: $27x = 117 + 63y$

$-63y + 27x = 117$ ——— (4) } step ②

$(3) + (4)$: $(63y + 14x) + (-63y + 27x) = 35 + 117$

$$41x = 152$$
$$x = 3\frac{29}{41}$$

Sub $x = 3\frac{29}{41}$ into (1),

$$9y + 2\left(3\frac{29}{41}\right) = 5$$
$$9y = -\frac{99}{41}$$
$$y = -\frac{11}{41}$$

$$\therefore x = 3\frac{29}{41}, \quad y = -\frac{11}{41}$$

Steps

① Compare equations (1) and (3). Pick a variable and make its coefficient the same in both equations by finding the Lowest Common Multiple (LCM). In e.g.1, pick $2x$ and $3x$ → LCM is $6x$.

② Ensure that the variable you want to eliminate is on the same side in both equations. In e.g.2, to eliminate y, shift $63y$ to the left-hand side.

③ Eliminate $6x$ by subtracting (4) from (3). OR (3) from (4) in this example.

*Note: Sometimes we add to eliminate variable.

④ Solve for y.

⑤ Substitute y value into either (1) or (2) to find x

⑥ Check your answers by substituting x and y values back into the original equations

OR Calculator Hack!

menu → 5 → 1 → 2 → key equations → "="

Before you key in your equations, ensure that they are in this form : $\boxed{ax + by = c}$

For example, for this question:

$9y + 2x = 5$

$2x + 9y = 5$ ——— (1)

$3x = 13 + 7y$

$3x - 7y = 13$ ——— (2)

Key in (1) and (2). You will get $x = 3\frac{29}{41}$, $y = -\frac{11}{41}$

eg 1. Solve the following pair of simultaneous equations.

$$\frac{3}{4}x - \frac{2}{5}y = 7 \quad \text{———— (1)}$$

$$9 - \frac{1}{2}x = \frac{3}{5}y \quad \text{———— (2)}$$

LCM of 4 and 5

$(1) \times 20: \; 15x - 8y = 140$ ———— (3)

$(2) \times 10: \; 90 - 5x = 6y$

LCM of 2 and 5

$270 - 15x = 18y$ ———— (4)

$(3)+(4): \; (15x - 8y) + (270 - 15x) = 140 + 18y$

$-8y + 270 = 140 + 18y$

$26y = 130$

$y = 5$

Sub $y = 5$ into (3),

$15x - 8(5) = 140$

$15x = 180$

$x = 12$

$\therefore x = 12$ and $y = 5$

TIP

If you have fractional coefficients, multiply the entire equation by the lowest common multiple of all the denominators.

2) Problems in Real World Context

eg 1. The difference between two numbers is 8. When one-third of the first number is added to one-half of the second number, the result is 18. Given that the first number is greater than the second number, find the sum of the two numbers.

Let the larger number (first) be x and the smaller number (second) be y.

$x - y = 8$

$x = 8 + y$ ———— (1)

$\frac{1}{3}x + \frac{1}{2}y = 18$ $\times 6$ (LCM of 3 and 2)

$2x + 3y = 108$ ———— (2)

Sub (1) into (2),

$2(8+y) + 3y = 108$

$16 + 2y + 3y = 108$

$5y = 108 - 16$

$y = 18.4$

Sub $y = 18.4$ into (1),

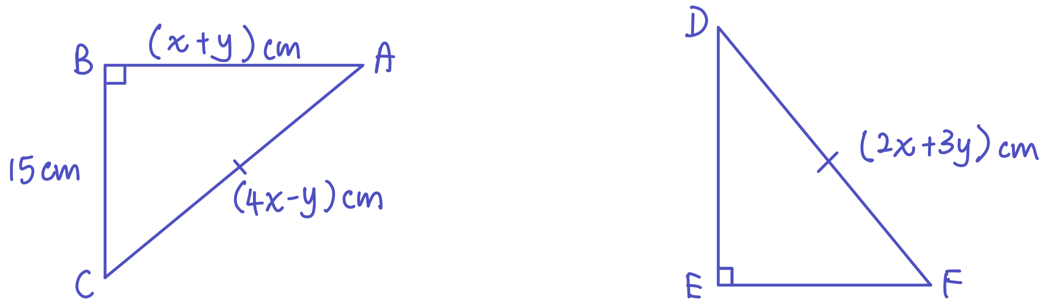
$x = 8 + 18.4$

$= 26.4$

Sum $= 18.4 + 26.4$

$= 44.8$

eg. 2 In the figure below, $AC = (4x-y)$ cm, $AB = (x+y)$ cm, $BC = 15$ cm and $DF = (2x+3y)$ cm. The area of triangle ABC is $45 cm^2$. Given that $AC = DF$, form two equations and find the length of DF.



Since $AC = DF$,

$$4x - y = 2x + 3y$$
$$2x = 4y$$
$$x = 2y \quad \text{——— (1)}$$

Area of $\triangle ABC = 45$

$$\frac{1}{2}(15)(x+y) = 45$$
$$15(x+y) = 90$$
$$x + y = 6 \quad \text{——— (2)}$$

Sub (1) into (2),

$$2y + y = 6$$
$$3y = 6$$
$$y = 2$$

Sub $y=2$ into (1),

$$x = 2(2)$$
$$= 4$$

$$DF = 2x + 3y$$
$$= 2(4) + 3(2)$$
$$= 14 \text{ cm}$$