

CHAPTER 1

AM

Quadratic Functions

1) Complete the square (Changing $y = ax^2 + bx + c$ to $y = a(x-h)^2 + k$)

$$\begin{aligned} x^2 + bx + c &= x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \\ &= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \end{aligned}$$

"number in front of x^2 "

* coefficient of x^2 must be before you start completing the square.

Example 1 : Express the following in the form of $a(x-h)^2 + k$.

Steps		Example : $x^2 + 10x$	$b =$
1)	Copy the x^2 term and bx term. Add $\left(\frac{b}{2}\right)^2$, subtract $\left(\frac{b}{2}\right)^2$ Copy the constant.	=	
2)	First three terms becomes $\left(x + \frac{b}{2}\right)^2$	=	
3)	Combine the constants. 	=	

Example 2 : When coefficient of x^2 is NOT +1.

Steps		Example : $-3x^2 + 5x + 7$	$b =$
1)	Factorise the coefficient of x^2 . Leave the constant alone.	$-3x^2 + 5x + 7$ = complete the square for this bracket. Everything else outside the bracket stays the same.	$b =$
2)	Copy the x^2 term and bx term. Add $\left(\frac{b}{2}\right)^2$, subtract $\left(\frac{b}{2}\right)^2$ Copy the constant.	=	
3)	First three terms becomes $\left(x + \frac{b}{2}\right)^2$	=	
4)	Expand the outer bracket.	=	
5)	Combine the constants. 	=	

2) Graph Sketching

Key pieces of information to find before sketching a graph

① Shape $\longrightarrow$ look at coefficient of $x^2 \longrightarrow$ If positive : $\cup$ (smiley face $\smile$)
If negative : $\cap$ (sad face $\frown$)

② x -intercept(s) $\longrightarrow$ When $y = 0$

③ y -intercept $\longrightarrow$ When $x = 0$

④ Turning point coordinates $\longrightarrow$ From complete the square form : (h, K)
(minimum/maximum)

x -coordinate :

Add the two x -intercepts
 $\frac{\quad}{2}$

$\longrightarrow$ y -coordinate :

Sub x -coordinate into equation.

optional

⑤ Line of symmetry : $x =$

Forms of quadratic equation		Which pieces of information can be obtained directly
General	$y = ax^2 + bx + c$ $y = 9x^2 - 8x + 13$	$\checkmark$ Shape $\longrightarrow$ $\checkmark$ y -intercept = $c =$
Factorised	$y = \pm a(x-p)(x-q)$ $y = 2(x+3)(x-8)$	$\checkmark$ Shape $\longrightarrow$ $\checkmark$ x -intercept = $(p, q) =$
Complete the square	$y = \pm a(x-h)^2 + k$ $y = -5(x+7)^2 + 15$	$\checkmark$ Shape $\longrightarrow$ $\checkmark$ Turning point coordinates = $(h, K) =$ (maximum)

Eg 1 : (a) Express $4x^2 - 16x + 3$ in the form of $a(x-h)^2 + k$ and state the coordinates of the turning point of the graph of $f(x) = 4x^2 - 16x + 3$
(b) Hence, sketch the graph and state its maximum or minimum value.

(a)

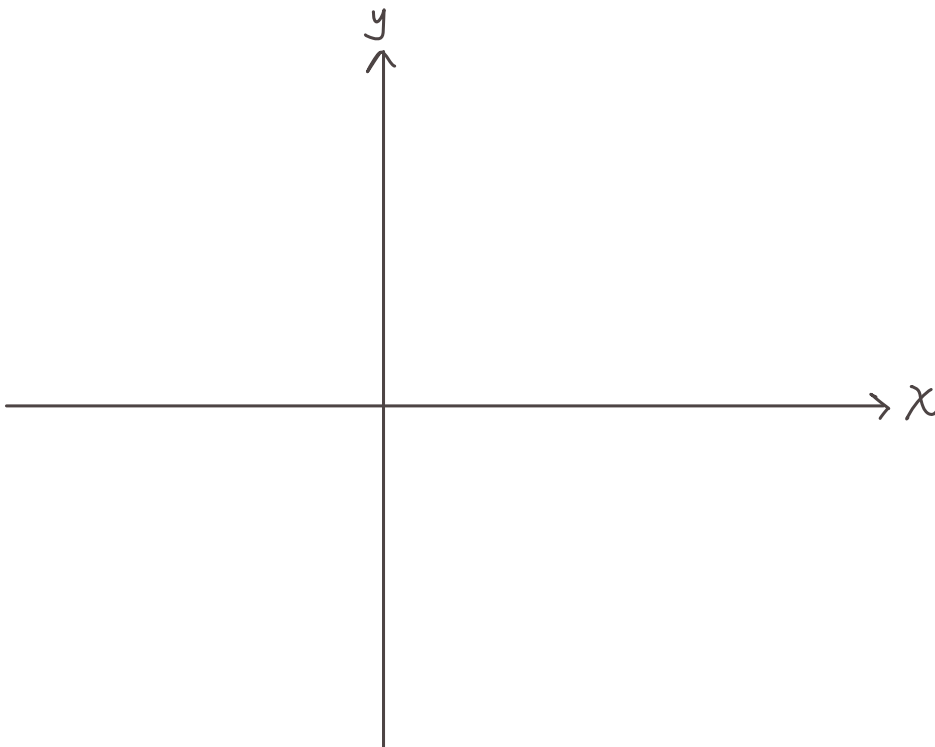
(b)

① Shape :

② x -intercepts =

③ y -intercept =

④ max
point =



3) Understanding the Question

$$y = a(x-h)^2 + k$$

When the question asks for...

- Maximum or minimum **value** $\longrightarrow$ _____ of the turning point =
- Turning point **coordinates** $\longrightarrow$ write as _____.

4) Whether curve lies completely above / below x -axis / a particular value

① Complete the square.

② State conditions and conclude:

Since the coefficient of $x^2 =$ which is , and the value of the curve is , then the curve lies completely the x -axis.

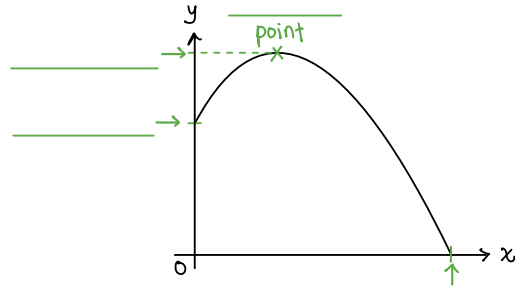
Eg: Explain why the value of $y = 2x^2 + 3x - 2$ will never be less than -4 .

Chap 1 & 2 Guideline

- Maximum / Minimum value or coordinate
 $\longrightarrow$
- Whether the curve lies completely above or below x -axis / curve is always positive or negative
 $\longrightarrow$
- Number of roots / intersections
 $\longrightarrow$

5) Application Questions

TIP Sketch the curve (if not given) to help visualise the problem.



Eg: N2023/P1/Q12

A ball is thrown vertically upwards. Its height, h m, above the ground at time t seconds after being thrown is given by the formula $h = 1.75 + 5t - 5t^2$.

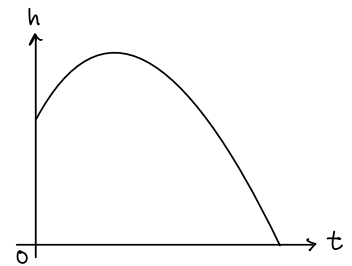
(a) State the height above the ground from which the ball is thrown.

(b) Express h in the form of $a + b(t + c)^2$ where a , b and c are constants to be determined.

(c) Hence, state the maximum height attained by the ball and the time at which this occurs.

extended:

(d) Find the height of the ball 0.7 seconds after it has been thrown, stating whether it is moving upwards or downwards.



extended:

(e) Find the duration for which the ball is at least 2.5 m above the ground.

